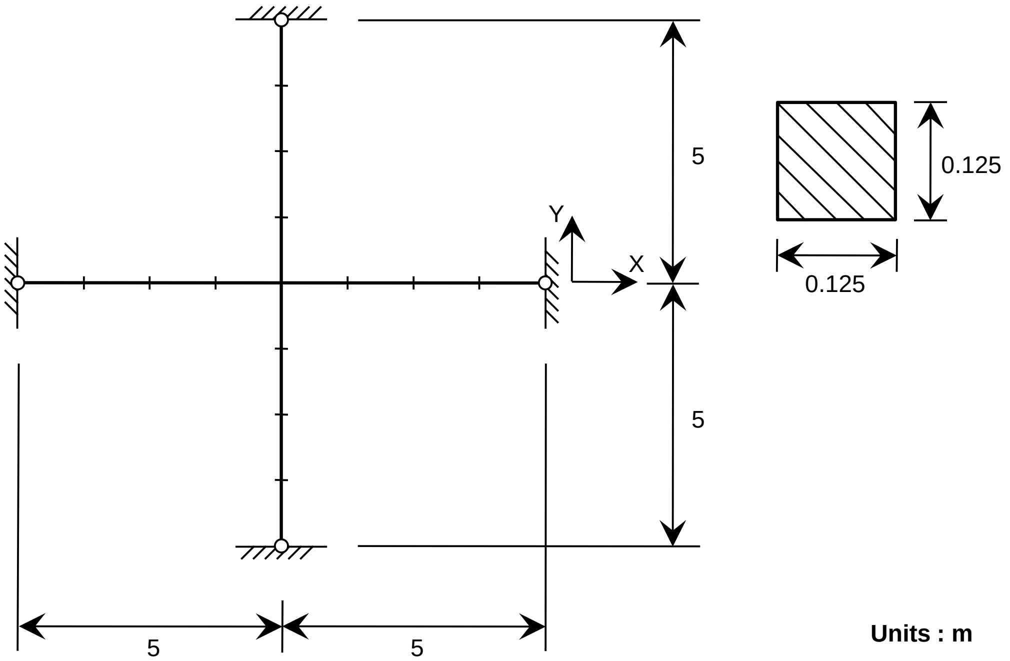
4.1 Pin-ended Cross

REFERENCE	NAFEMS[1]
ELEMENTS	Beam elements
MODEL FILENAME	Dynamics01.gts

Figure 4.1.1 shows a pin-ended cross model. The structure is modeled with 4 beam elements for each arm. Eigenvalue analysis is carried out. The natural frequencies of the lowest modes of which the vibration mode shapes are depicted in Figure 4.1.2 are compared with the reference values given in the NAFEMS benchmarks.

Figure 4.1.1
Pin-ended cross model



Material data	Young's modulus Unit weight	*E = 200 MPa* γ = *78.4532 kN/m*^{3}
Section property	Square cross-section	*0.125 m x 0.125 m*

Figure 4.1.2
Vibration mode shapes

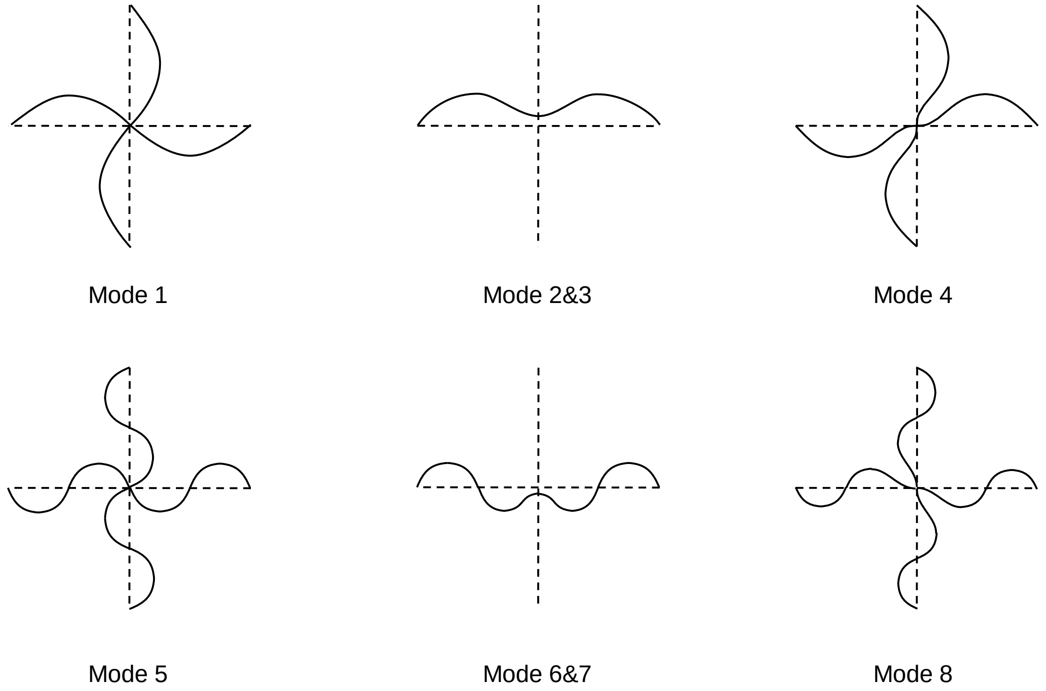


Table 4.1.1 Natural frequencies in Hz obtained using beam elements

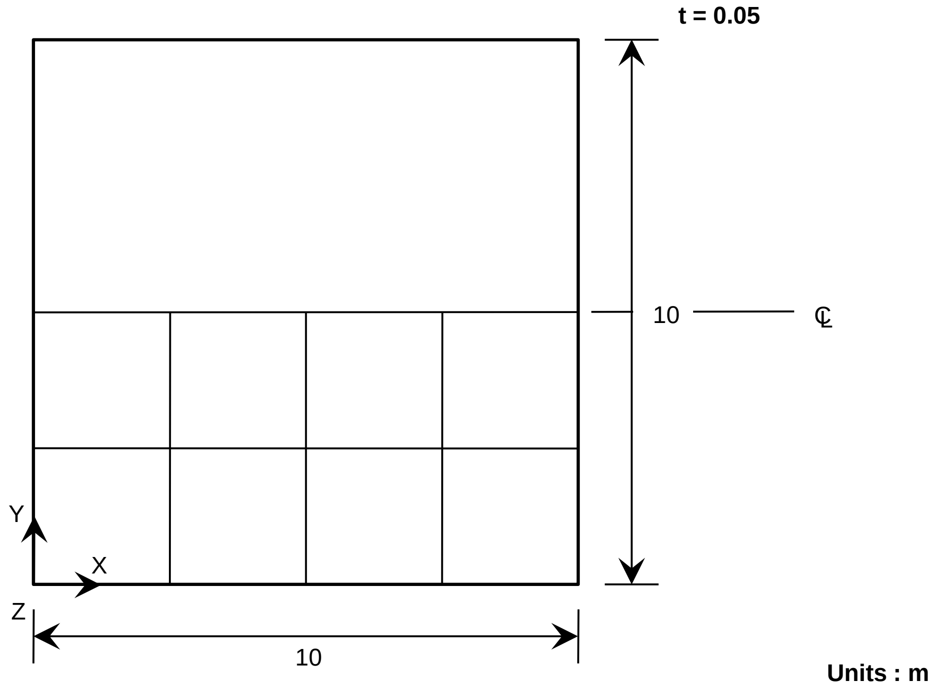
Mode Number		1	2, 3	4	5	6, 7	8
Reference		*11.336*	*17.709*	*17.709*	*45.345*	*57.390*	*57.390*
Element type	Number of elements						
Beam-2	4 per arm	*11.338*	*17.689*	*17.717*	*45.483*	*57.371*	*57.690*

4.2 Thin Square Cantilever Plate

REFERENCE	NAFEMS[1]
ELEMENTS	Shell elements, solid elements
MODEL FILENAME	Dynamics02(CaseA).gts Dynamics02(CaseB).gts

Figure 4.2.1 shows a thin square cantilever plate model. One half of the model is discretized using shell and solid elements. Depending on the applied boundary conditions, eigenvalue analyses of two sub-cases are conducted; Case A considers only the symmetric modes and Case B considers only the anti-symmetric vibration modes.

Figure 4.2.1
Thin square cantilever plate model



case A $x = y = R_z = 0$ at all nodes,
$z = R_y = R_x = 0$ along y-axis, $R_x = 0$ along y = 5m

case B $x = y = R_z = 0$ at all nodes,
$z = R_y = R_x = 0$ along y-axis

Material data	Young's modulus	*E = 200 MPa*
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532\ kN/m^3$
Section property	Thickness	*t = 0.05 m*

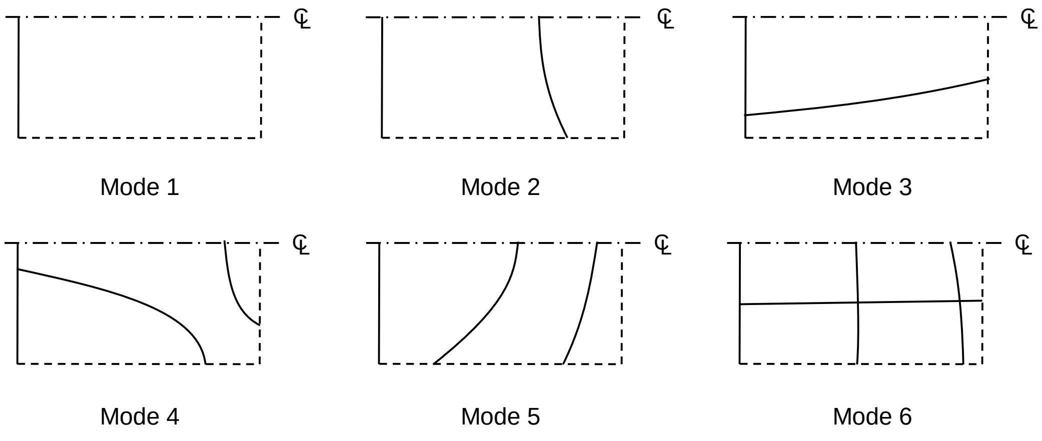


Figure 4.2.2
Vibration mode shapes
(Case A)

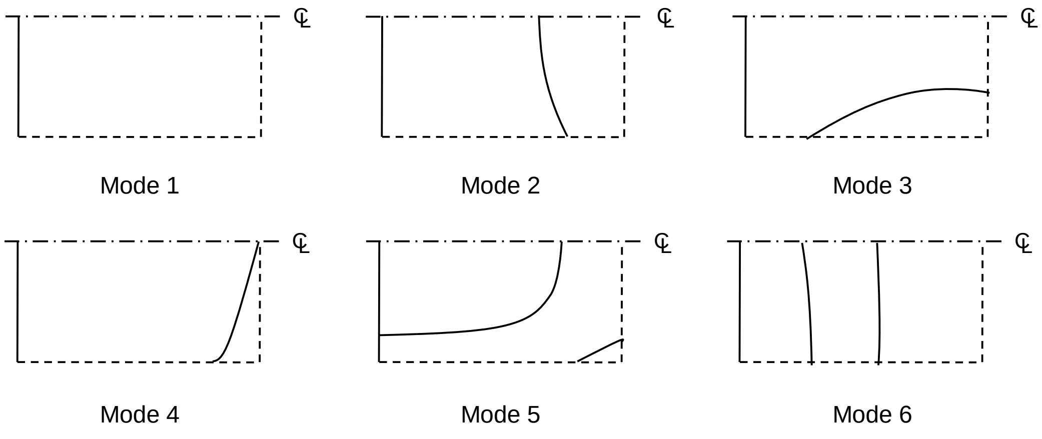


Figure 4.2.3
Vibration mode shapes
(Case B)

Table 4.2.1 Natural frequencies in Hz obtained using shell elements; symmetric vibration modes (Case A)

Mode Number		1	2	3	4	5	6
Reference		*0.421*	*2.582*	*3.306*	*6.555*	*7.381*	*11.402*
Element type	Number of elements						
TRIA-3	64	*0.418*	*2.639*	*3.369*	*6.951*	*7.974*	*13.032*
QUAD-4	8x4	*0.418*	*2.615*	*3.337*	*6.752*	*7.905*	*12.681*
TRIA-6	16	*0.418*	*2.637*	*3.290*	*6.748*	*7.809*	*12.636*
QUAD-8	4x2	*0.419*	*2.569*	*3.281*	*6.566*	*7.497*	*12.129*

Table 4.2.2 Natural frequencies in Hz obtained using solid elements; symmetric vibration modes (Case A)

Mode Number		1	2	3	4	5	6
Reference		*0.421*	*2.582*	*3.306*	*6.555*	*7.381*	*11.402*
Element type	Number of elements						
HEXA-8	8x4x1	*0.419*	*2.656*	*3.353*	*6.747*	*8.251*	*12.654*
PENTA-15	16	*0.436*	*2.909*	*4.043*	*9.401*	*10.253*	*21.691*
HEXA-20	4x2x1	*0.431*	*2.664*	*3.426*	*7.617*	*7.885*	*14.343*

Table 4.2.3 Natural frequencies in Hz obtained using shell elements; anti-symmetric vibration modes (Case B)

Mode Number		1	2	3	4	5	6
Reference		*1.029*	*3.753*	*7.730*	*8.561*	*N/A*	*N/A*
Element type	Number of elements						
TRIA-3	64	*1.017*	*3.788*	*8.201*	*9.111*	*12.246*	*17.365*
QUAD-4	8x4	*1.026*	*3.807*	*8.220*	*9.194*	*11.974*	*18.004*
TRIA-6	16	*1.022*	*3.744*	*7.849*	*8.732*	*11.292*	*16.426*
QUAD-8	4x2	*1.024*	*3.730*	*7.625*	*8.606*	*11.167*	*16.807*

Table 4.2.4 Natural frequencies in Hz obtained using solid elements; anti-symmetric vibration modes (Case B)

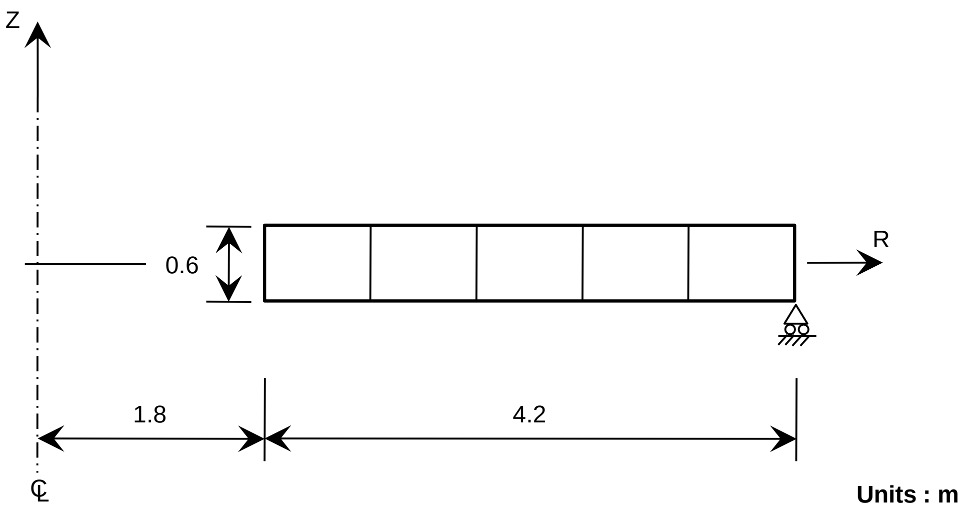
Mode Number		1	2	3	4	5	6
Reference		*1.029*	*3.753*	*7.730*	*8.561*	*N/A*	*N/A*
Element type	Number of elements						
HEXA-8	8x4x1	*1.029*	*3.840*	*8.356*	*9.416*	*11.977*	*18.248*
PENTA-15	16	*1.197*	*4.880*	*10.618*	*13.128*	*22.087*	*27.706*
HEXA-20	4x2x1	*1.043*	*3.830*	*8.237*	*9.102*	*14.733*	*17.537*

4.3 Simply Supported Thick Annular Plate (Axisymmetric)

REFERENCE	NAFEMS[1]
ELEMENTS	Axisymmetric elements, solid elements
MODEL FILENAME	Dynamics03(AXISYM).gts Dynamics03(SOLID).gts

A simply-supported thick annular plate problem is depicted in Figure 4.3.1. As an axisymmetric problem, the annular plate is modeled using axisymmetric elements and solid elements with the boundary condition to enforce axisymmetry. The natural frequencies of the lowest modes of which the vibration mode shapes are depicted in Figure 4.3.2 are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.3.1
Simply-supported axisymmetric annular plate model



Material data	Young's modulus Poisson's ratio Unit weight	$E = 200\ MPa$ $\nu = 0.3$ $\gamma = 78.4532\ kN/m^3$
Section property	Thickness	$t = 0.6\ m$

Figure 4.3.2
Vibration mode shapes

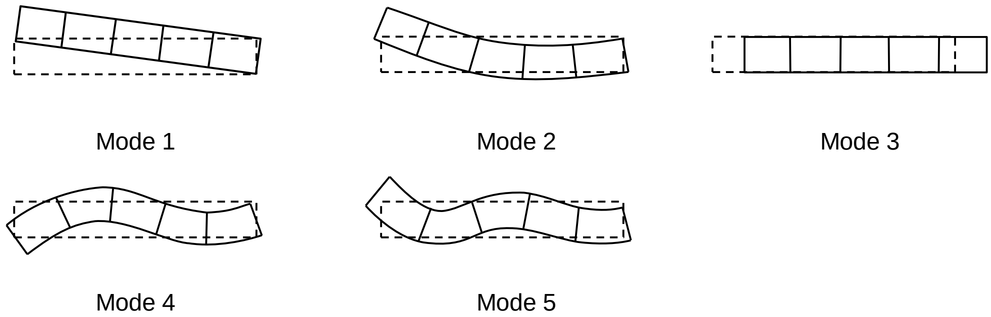


Table 4.3.1 Natural frequencies in Hz obtained using axisymmetric elements

Mode Number		1	2	3	4	5
Reference		*18.583*	*140.15*	*224.16*	*358.29*	*629.19*
Element type	Number of elements					
QUADX-4	5	*18.569*	*147.47*	*223.63*	*441.20*	*695.50*
TRIAX-3	10	*20.428*	*223.39*	*291.56*	*684.85*	*731.18*
QUADX-8	5	*18.582*	*140.56*	*224.18*	*374.05*	*686.04*
TRIAX-6	10	*18.590*	*140.90*	*224.16*	*379.00*	*688.48*

Table 4.3.2 Natural frequencies in Hz obtained using solid elements

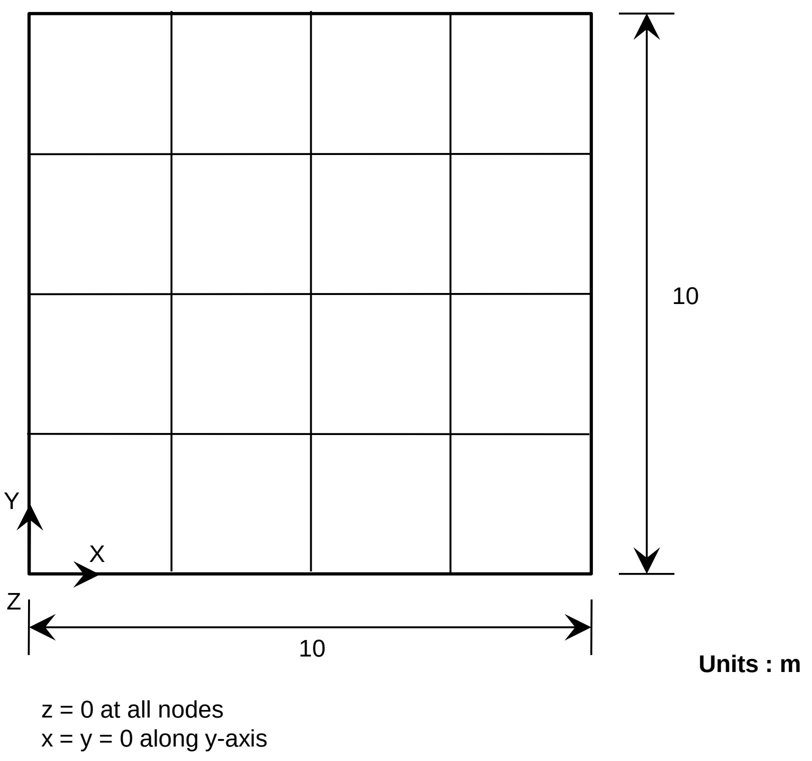
Mode Number		1	2	3	4	5
Reference		*18.583*	*140.15*	*224.16*	*358.29*	*629.19*
Element type	Number of elements					
TETRA-4	565	*20.910*	*159.43*	*224.52*	*413.64*	*688.22*
PENTA-6	120	*18.951*	*141.08*	*224.55*	*366.24*	*648.70*
HEXA-8	15x1x4	*18.600*	*139.20*	*224.22*	*362.81*	*645.35*
TETRA-10	59	*18.893*	*139.87*	*224.60*	*368.30*	*664.11*
PENTA-15	10	*18.658*	*135.18*	*224.06*	*344.42*	*598.98*
HEXA-20	5x1x1	*18.389*	*133.28*	*223.28*	*338.86*	*590.50*

4.4 Cantilevered Square Plane Stress

REFERENCE	NAFEMS[1]
ELEMENTS	Plane stress elements
MODEL FILENAME	Dynamics04.gts

Figure 4.4.1 shows a cantilevered square plane stress problem. Plane stress elements are employed to obtain the lowest vibration frequencies. The results are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.4.1
Cantilevered square plane stress model



Material data	Young's modulus	*E* = 200 MPa
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532\ kN/m^3$

Figure 4.4.2
Vibration mode shapes

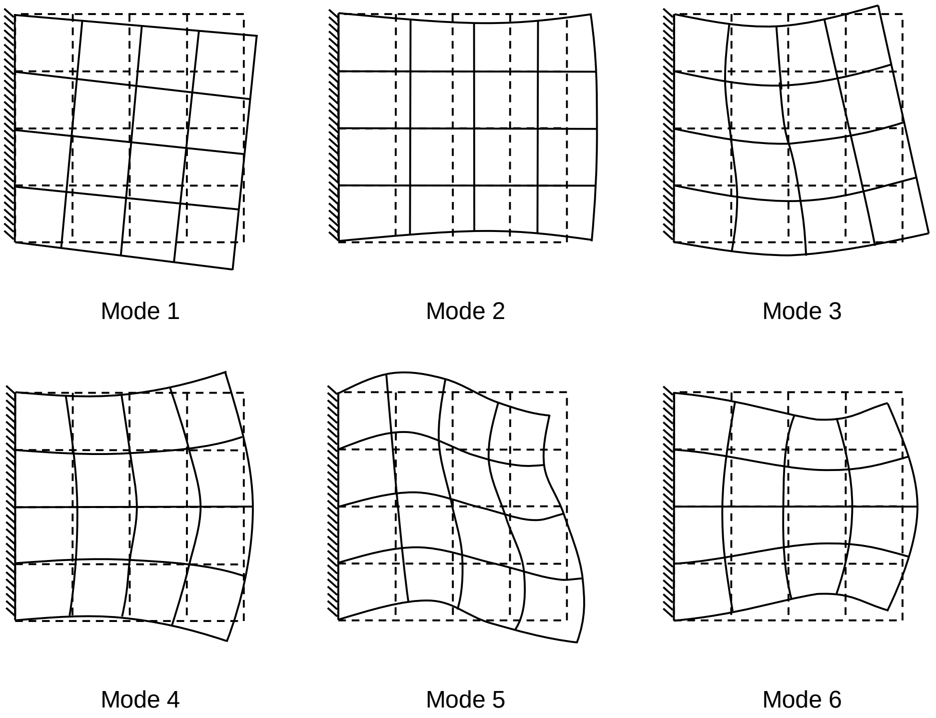


Table 4.4.1 Natural frequencies in Hz obtained using plane stress elements

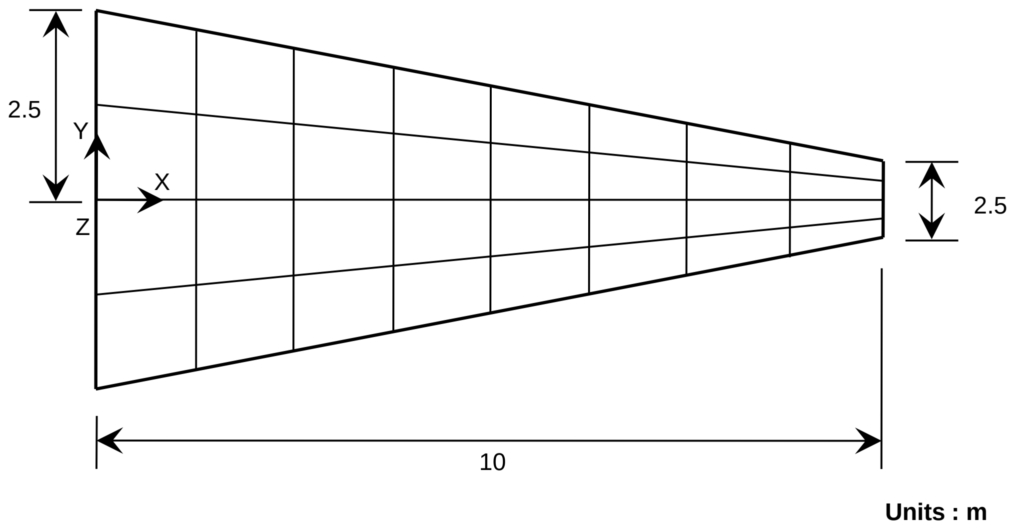
Mode Number		1	2	3	4	5	6
Reference		*52.404*	*125.69*	*140.78*	*222.54*	*241.41*	*255.74*
Element type	Number of elements						
TRIAX-3	128	*53.549*	*125.89*	*142.19*	*226.83*	*243.51*	*259.46*
QUADX-4	8x8	*52.426*	*125.59*	*139.48*	*214.51*	*239.81*	*252.03*
TRIAX-6	32	*52.646*	*125.87*	*141.76*	*225.67*	*243.90*	*257.64*
QUADX-8	4x4	*52.603*	*125.85*	*141.41*	*224.45*	*243.00*	*256.67*

4.5 Cantilevered Tapered Membrane

REFERENCE	NAFEMS[2]
ELEMENTS	Plane stress elements
MODEL FILENAME	Dynamics05.gts

Figure 4.5.1 shows a cantilevered tapered membrane problem. Plane stress elements are employed to obtain the lowest vibration frequencies. The results are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.5.1
Cantilevered tapered membrane model



z = 0 at all nodes
x = y = 0 along y-axis

Material data	Young's modulus	E = 200 MPa
	Poisson's ratio	ν = 0.3
	Unit weight	γ = 78.4532 kN/m^3

Figure 4.5.2
Vibration mode shapes

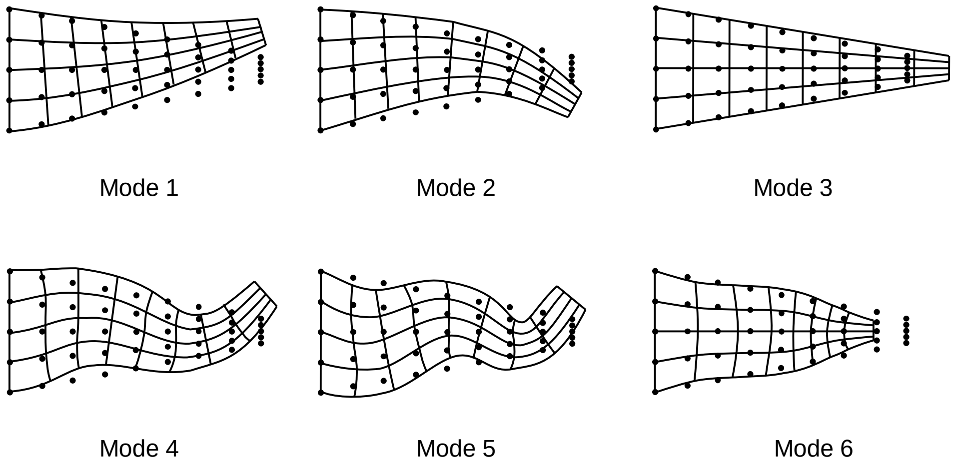


Table 4.5.1 Natural frequencies in Hz obtained using plane stress elements

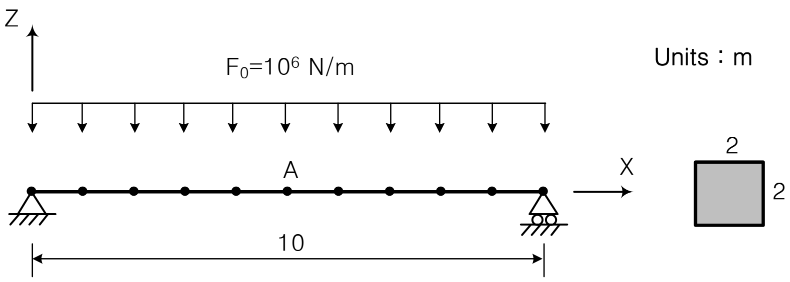
Mode Number		1	2	3	4	5	6
Reference		*44.623*	*130.03*	*162.70*	*246.05*	*379.90*	*391.44*
Element type	Number of elements						
TRIA-3	128	*45.643*	*134.53*	*162.92*	*258.37*	*393.48*	*404.95*
QUAD-4	8x8	*44.647*	*131.04*	*162.80*	*250.33*	*391.54*	*393.10*
TRIA-6	32	*44.645*	*130.19*	*162.72*	*247.05*	*383.47*	*391.58*
QUAD-8	4x4	*44.632*	*130.11*	*162.71*	*246.48*	*381.58*	*391.53*

4.6 Deep Simply Supported Beam

REFERENCE	NAFEMS[3]
ELEMENTS	Beam elements, solid elements
MODEL FILENAME	Dynamics06(Direct).gts Dynamics06(Modal).gts

A dynamic system consisting of a simply supported beam is shown in Figure 4.6.1. Two types of time-variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings, (2) a transient step load. Each load is applied in the transverse direction of the beam and uniformly distributed spatially. The peak displacement and stress are obtained in the time domain. The results are obtained at the mid-span using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.6.1 - 4.6.2. The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.6.1
Simply supported deep beam model



Material data	Young's modulus	*E = 200 MPa*
	Poisson's ratio	ν = 0.3
	Unit weight	γ = 78.4532 kN/m^3
Section property	Rectangular cross-section	*2 m x 2 m*
Forcing functions	Periodic	$F = F_0\left(\sin\omega t - \sin 3\omega t\right)$ (where $w = 2\pi f,\ f = 20Hz$)
	Transient	$F = F_0,\ t > 0$

** Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.*

Table 4.6.1 Peak responses of beam subjected to periodic forcing function

		Peak u_Z^A [*mm*]		Peak σ^A [*MPa*]	
Reference		*0.951*		*17.1*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*
BEAM-2	10	*0.955*	*0.955*	*17.5*	*17.4*
HEXA-8	10	*0.962*	*0.962*	*17.5*	*17.3*
PENTA-6	10x4	*0.964*	*0.965*	*17.3*	*18.1*
HEXA-20	5	*0.945*	*0.944*	*17.1*	*17.0*
PENTA-15	5x4	*0.945*	*0.945*	*16.9*	*16.9*
PYRAM-13	5x6	*0.944*	*0.945*	*17.5*	*17.5*

Table 4.6.2 Peak responses of beam subjected to transient step load

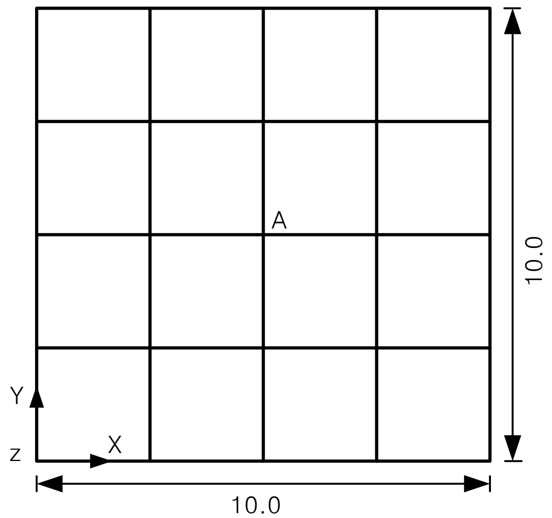
		Peak u_Z^A [*mm*]		Peak time [*sec*]		Peak σ^A [*MPa*]		Static u_Z^A [*mm*]	
Reference		*1.043*		*0.0117*		*18.76*		*0.538*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*
BEAM-2	10	*1.044*	*1.043*	*0.0117*	*0.0116*	*18.51*	*18.51*	*0.537*	*0.537*
HEXA-8	10	*1.012*	*1.1012*	*0.0116*	*0.0116*	*18.03*	*18.03*	*0.521*	*0.521*
PENTA-6	10x4	*0.946*	*0.945*	*0.0112*	*0.0112*	*17.40*	*17.31*	*0.487*	*0.487*
HEXA-20	5	*1.019*	*1.017*	*0.117*	*0.0116*	*18.06*	*17.96*	*0.524*	*0.525*
PENTA-15	5x4	*1.023*	*1.022*	*0.117*	*0.0116*	*18.05*	*18.04*	*0.527*	*0.527*
PYRAM-13	5x6	*1.014*	*1.013*	*0.117*	*0.0116*	*17.99*	*17.88*	*0.523*	*0.522*

4.7 Simply Supported Thin Square Plate

REFERENCE	NAFEMS[3]
ELEMENTS	Shell elements
MODEL FILENAME	Dynamics07(Direct).gts Dynamics07(Modal).gts

A dynamic system consisting of a thin square cantilever plate is shown in Figure 4.7.1. Two types of time variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings, (2) a transient step load. Each load is uniformly distributed spatially and applied in the transverse direction of the plate. The peak displacement and stress are obtained either in the frequency domain or the time domain. The results are obtained at the center point using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.7.1-4.7.2. The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.7.1
Simply supported thin plate model



F_0=100 N/m^2
x = y = Rz = 0 at all nodes
Rx = Ry = 0 along 4 edges

Units: m

Material data	Young's modulus	*E* = 200 MPa
	Poisson's ratio	ν = 0.3
	Unit weight	γ = 78.4532 kN/m^3
	Mass proportional damping	α = 0.229 sec^{-1}
	Stiffness proportional damping	β = 1.339x10^{-3} sec
	Modal damping	ξ = 0.02
Section property	Thickness	*t* = 0.05 m
Forcing functions	(1) Periodic	$F = F_0\left(\sin \omega t - \sin 3\omega t\right)$ (where $w = 2\pi f$, $f = 20Hz$)
	(2) Transient	$F = F_0$, $t > 0$

* Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.

Table 4.7.1 Peak responses of thin plate subjected to periodic forcing function

		Peak u_Z^A [*mm*]		Peak σ^A [*MPa*]	
Reference		*2.863*		*2.018*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*
QUAD-4	8x8	*2.913*	*2.916*	*2.072*	*2.077*
TRIA-3	128	*2.881*	*2.883*	*2.068*	*2.074*
QUAD-8	4x4	*2.884*	*2.885*	*2.274*	*2.277*
TRIA-6	32	*2.890*	*2.892*	*2.177*	*2.177*

Table 4.7.2 Peak responses of thin plate subjected to transient step load

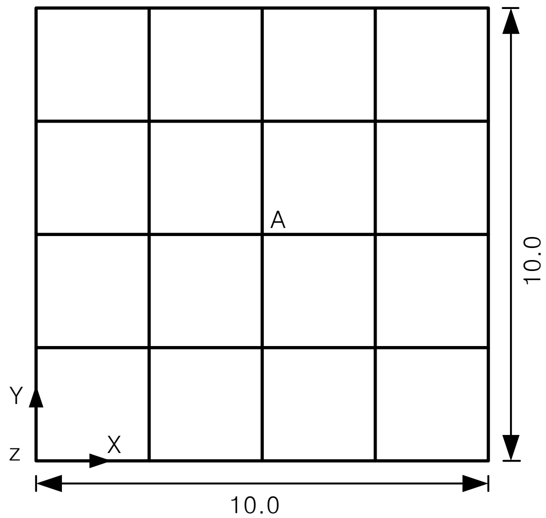
		Peak u_Z^A [*mm*]		Peak time [*sec*]		Peak σ^A [*MPa*]		Static u_Z^A [*mm*]	
Reference		*3.523*		*0.210*		*2.484*		*1.817*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*
QUAD-4	8x8	*3.474*	*3.487*	*0.210*	*0.211*	*2.376*	*2.425*	*1.770*	*1.770*
TRIA-3	128	*3.342*	*3.354*	*0.206*	*0.207*	*2.306*	*2.349*	*1.703*	*1.703*
QUAD-8	4x4	*3.472*	*3.472*	*0.214*	*0.216*	*2.522*	*2.541*	*1.774*	*1.774*
TRIA-6	32	*3.485*	*3.473*	*0.210*	*0.207*	*2.355*	*2.321*	*1.786*	*1.786*

4.8 Simply Supported Thick Square Plate

REFERENCE	NAFEMS[3]
ELEMENTS	Shell elements, solid elements
MODEL FILENAME	Dynamics08(Direct).gts Dynamics08(Modal).gts

A dynamic system consisting of a thick square cantilever plate is shown in Figure 4.8.1. Two types of time-variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings and (2) a transient step load. Each load is uniformly distributed spatially and applied in the transverse direction of the plate. The peak displacement and stress are obtained in the time domain. The results are obtained at the center point using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.8.1-4.8.2.The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.8.1
Simply supported thick plate model



F_0=10^6 N/m^2
x = y = Rz = 0 at all nodes
Rx = Ry = 0 along 4 edges

Units: m

Material data	Young's modulus	*E = 200 MPa*
	Poisson's ratio	ν = 0.3
	Unit weight	γ = 78.4532 kN/m^3
	Mass proportional damping	α = 0.229 sec^{-1}
	Stiffness proportional damping	β = 1.339x10^{-3} sec
	Modal damping	ξ = 0.02
Section property	Thickness	*t = 1.0 m*
Forcing functions	(1) Periodic	$F = F_0\left(\sin\omega t - \sin 3\omega t\right)$ (where $w = 2\pi f$, $f = 20Hz$)
	(2) Transient	$F = F_0,\ t > 0$

* Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.

Table 4.8.1 Peak responses of thin plate subjected to periodic forcing function

		Peak u_Z^A [*mm*]		Peak σ^A [*MPa*]	
Reference		*4.929*		*67.67*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*
QUAD-4	8x8	*5.294*	*5.298*	*72.35*	*72.46*
TRIA-3	128	*5.419*	*5.422*	*73.26*	*73.32*
QUAD-8	4x4	*5.058*	*5.061*	*72.13*	*72.19*
TRIA-6	32	*5.022*	*5.024*	*70.91*	*70.98*
HEXA-8	96	*5.344*	*53.37*	*70.19*	*70.87*
HEXA-20	48	*5.048*	*5.050*	*76.49*	*76.57*
PENTA-15	64	*5.163*	*5.165*	*73.06*	*73.12*

Table 4.8.2 Peak responses of thin plate subjected to transient step load

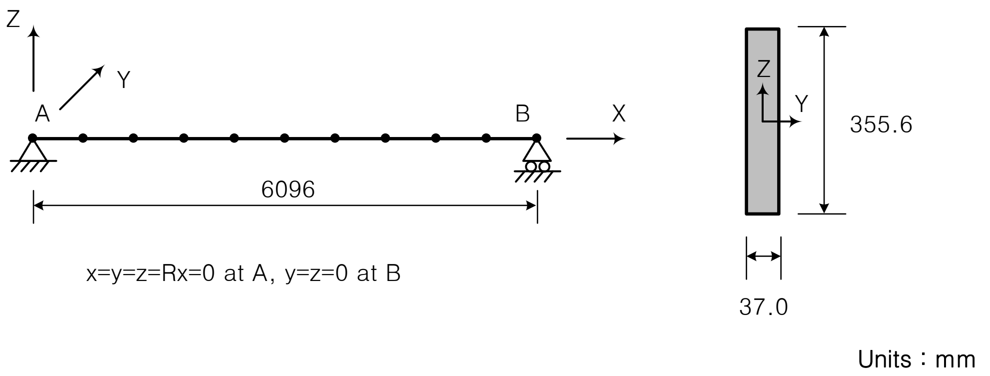
		Peak u_Z^A [*mm*]		Peak time [*sec*]		Peak σ^A [*MPa*]		Static u_Z^A [*mm*]	
Reference		*4.524*		*0.0108*		*62.11*		*2.333*	
Element type	Number of elements	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*	*Direct*	*Modal*
QUAD-4	8x8	*4.563*	*4.547*	*0.0107*	*0.0105*	*58.06*	*57.40*	*2.339*	*2.339*
TRIA-3	128	*4.388*	*4.372*	*0.0105*	*0.105*	*55.66*	*54.84*	*2.249*	*2.249*
QUAD-8	4x4	*4.827*	*4.845*	*0.0110*	*0.0109*	*66.71*	*67.81*	*2.423*	*2.423*
TRIA-6	32	*4.773*	*4.778*	*0.0109*	*0.0108*	*63.68*	*64.06*	*2.422*	*2.421*
HEXA-8	96	*4.505*	*4.489*	*0.0108*	*0.0109*	*56.39*	*55.80*	*2.310*	*2.310*
HEXA-20	48	*4.467*	*4.458*	*0.0107*	*0.0106*	*62.85*	*62.35*	*2.283*	*2.283*
PENTA-15	64	*4.354*	*4.343*	*0.0106*	*0.0103*	*57.57*	*57.39*	*2.229*	*2.229*

4.9 Response Spectrum of a Simply Supported Beam

REFERENCE	Biggs, J. M.[4]
ELEMENTS	Beam elements, shell elements, solid elements
MODEL FILENAME	Dynamics09.gts

Figure 4.9.1 shows a simply-supported beam problem with a rectangular cross section. The response spectrum defined in Table 4.9.1 is applied in the vertical direction of the beam at its ends. Response spectrum analyses are carried out with various types of elements. Damping is assumed to be zero. The maximum displacement, bending moment and stress at the mid-span are obtained using the square root of the sum of squares (SRSS) summation method.

Figure 4.9.1
Simply supported beam model



Material data	Young's modulus Unit Weight	*E = 206.8 GPa* γ *= 1027.05046 kN/m³*
Section property	Rectangular cross-section	*2 m x 2 m*

Table 4.9.1 Response spectra definition(unit : m)

Frequency [*Hz*]		*5.000*	*6.000*	*6.098*	*7.000*	*8.000*
Period [*sec*]		*0.2000*	*0.1667*	*0.1640*	*0.1429*	*0.1250*
Type (scale factor)	Displacement(1.0)	*0.0199*	*0.0115*	*0.0110*	*0.0072*	*0.0056*
	Velocity(1.0)	*0.6248*	*0.4339*	*0.4201*	*0.3188*	*0.2837*
	Acceleration(0.5)	*39.258*	*32.716*	*32.190*	*28.042*	*28.521*

Table 4.9.2 Response spectrum analysis results obtained using beam elements

Result at mid-span		Displacement [*mm*]	Stress [*MPa*]	Moment $\times 10^5$ [*Nm*]
Reference		*14.2*	*140.4*	*1.095*
Element type	Spectra type			
BEAM	Displacement	*14.2*	*138.4*	*1.079*
	Velocity	*14.1*	*138.1*	*1.077*
	Acceleration	*14.1*	*138.1*	*1.077*

Table 4.9.3 Response spectrum analysis results obtained using shell elements

Result at mid-span		Displacement [*mm*]	Stress [*MPa*]
Reference		*14.2*	*140.4*
Element type	Spectra type		
QUAD-4	Displacement	*13.8*	*132.4*
	Velocity	*13.7*	*132.0*
	Acceleration	*13.7*	*132.0*
TRIA-3	Displacement	*13.9*	*134.6*
	Velocity	*13.9*	*134.2*
	Acceleration	*13.9*	*134.2*
QUAD-8	Displacement	*14.2*	*138.4*
	Velocity	*14.1*	*138.1*
	Acceleration	*14.1*	*138.1*
TRIA-6	Displacement	*14.2*	*138.8*
	Velocity	*14.2*	*138.6*
	Acceleration	*14.2*	*138.6*

Table 4.9.4 Response spectrum analysis results obtained using solid elements

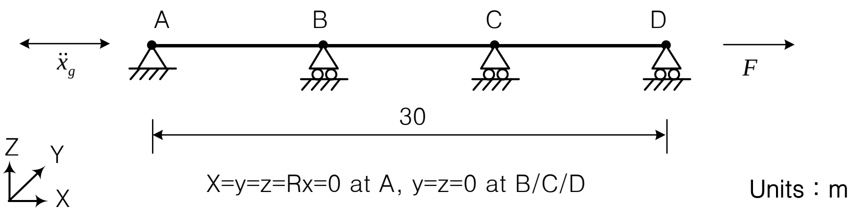
Result at mid-span		Displacement [*mm*]	Stress [*MPa*]
Reference		*14.2*	*140.4*
Element type	Spectra type		
HEXA-8	Displacement	*13.7*	*132.3*
	Velocity	*13.7*	*131.9*
	Acceleration	*13.7*	*131.9*
HEXA-20	Displacement	*14.1*	*138.3*
	Velocity	*14.1*	*138.0*
	Acceleration	*14.1*	*138.0*
PENTA-15	Displacement	*14.1*	*137.6*
	Velocity	*14.1*	*137.3*
	Acceleration	*14.1*	*137.3*

4.10 Linear Dynamic Analyses of a Truss

REFERENCE	Chopra, A. K.[5]
ELEMENTS	Truss elements, plane stress elements, solid elements
MODEL FILENAME	Dynamics10.gts

A one-dimensional truss model with three members is depicted in Figure 4.10.1. Truss, plane stress and solid elements are used to model the problem. The results are obtained at the point D using modal methods including transient analysis and response spectrum. Comparison is made with the closed form solutions.

Figure 4.10.1
Simply supported beam model



Material data	Young's modulus Unit weight	*E = 5 Pa* *γ = 0.108961688 N/m^3*
Section property	Cross-section Area	*A = 2.0 m^2*
Analysis condition	Modal transient with tip load Modal transient with base acceleration Response spectrum	*F = 10 N, 10% damping* *x_R =1.0m/sec^2,10% damping* *displacement spectra, 2% damping*

Figure 4.10.2
Displacement response spectra

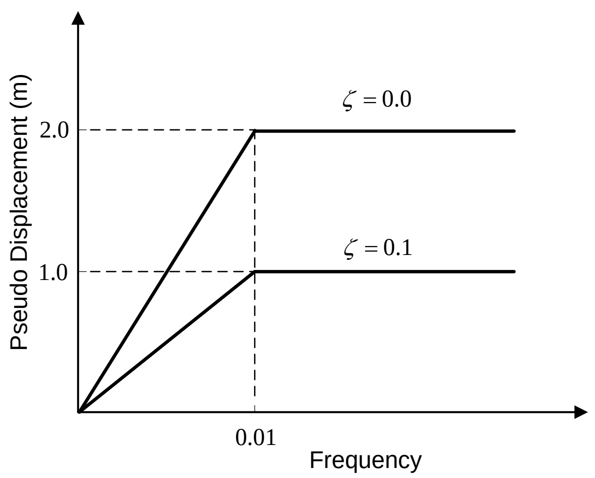


Table 4.10.1 Displacement and acceleration at point D using modal transient analysis with tip load

Result type	*Displacement* [*m*]			*Acceleration* [*m/sec*^{2}]		
Time step [*sec*]	*0.1*	*0.2*	*0.3*	*0.1*	*0.2*	*0.3*
Reference	*0.4387*	*1.686*	*3.598*	*81.42*	*66.71*	*48.06*
Element type						
TRUSS-2	*0.4387*	*1.686*	*3.598*	*81.42*	*66.71*	*48.06*
QUAD-4	*0.4387*	*1.686*	*3.598*	*81.42*	*66.71*	*48.06*
HEXA-8	*0.4387*	*1.686*	*3.598*	*81.42*	*66.71*	*48.06*

Table 4.10.2 Total displacement at point D using modal transient analysis with base acceleration

Result type	*Displacement* [*m*]		
Time step [*sec*]	*0.1*	*0.2*	*0.3*
Reference	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
Element type			
TRUSS-2	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
QUAD-4	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
HEXA-8	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}

Table4.10.3 Peak displacement at point D using response spectrum analysis with 2% modal damping ratio

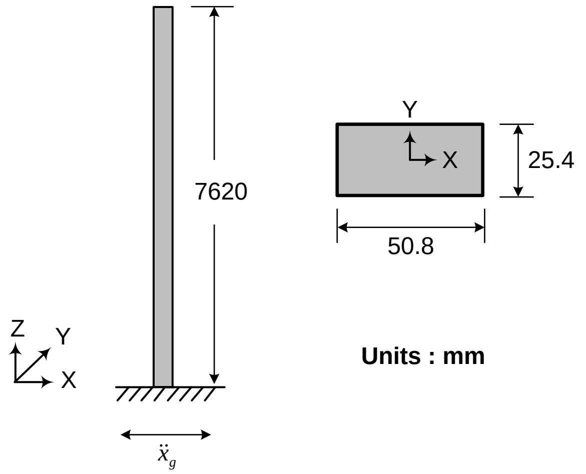
Result type	*Displacement* [*m*]				
Combination method	ABS	SRSS	TENP	NRL	CQC
Reference	*2.902*	*2.248*	*2.248*	*2.767*	*2.246*
Element type	*2.902*	*2.248*	*2.248*	*2.767*	*2.246*
TRUSS-2	*2.902*	*2.248*	*2.248*	*2.767*	*2.246*
QUAD-4	*2.902*	*2.248*	*2.248*	*2.767*	*2.246*
HEXA-8	*2.902*	*2.248*	*2.248*	*2.767*	*2.246*

4.11 Cantilever Subjected to Earthquake Motion

REFERENCE	Hiber, H. M. et al.[6] and Hurty, W.C. et al[7]
ELEMENTS	Beam elements
MODEL FILENAME	Dynamics11.gts

Figure 4.11.1 shows a vertical cantilever beam model. Seismic analyses are carried out in which the anchor point of the beam model is subjected to a prescribed motion. The time history of acceleration of the El Centro N-S as shown in Figure 4.11.2 is applied in the horizontal direction. The transient response of the structure is obtained using the direct and modal superposition methods. The maximum displacement and velocity at the top location is summarized in Table 4.11.1. Response spectrum analyses are also conducted based on the response spectra shown in Figures 4.11.3 and 4.11.4 using the ABS and SRSS combination methods.

Figure 4.11.1
Simply supported beam model



Material data	Young’s modulus Unit Weight	*E = 206.8 GPa* γ *= 78.4532 kN/m*^{3}
Section property	Rectangular cross-section	*50.8 mm x 25.4 mm*

Figure 4.11.2
El Centro N-S
acceleration history

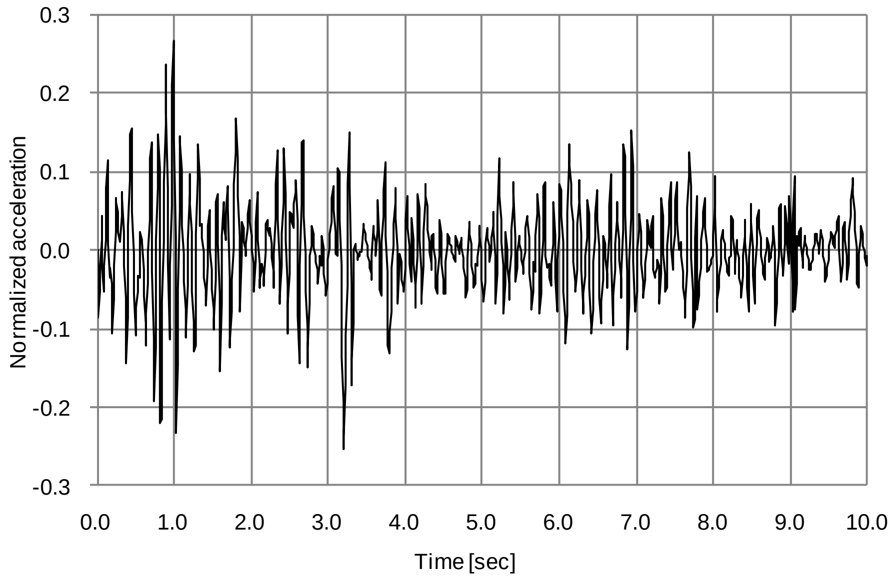


Figure 4.11.3
Displacement spectra
for the period range
0.03~10 sec

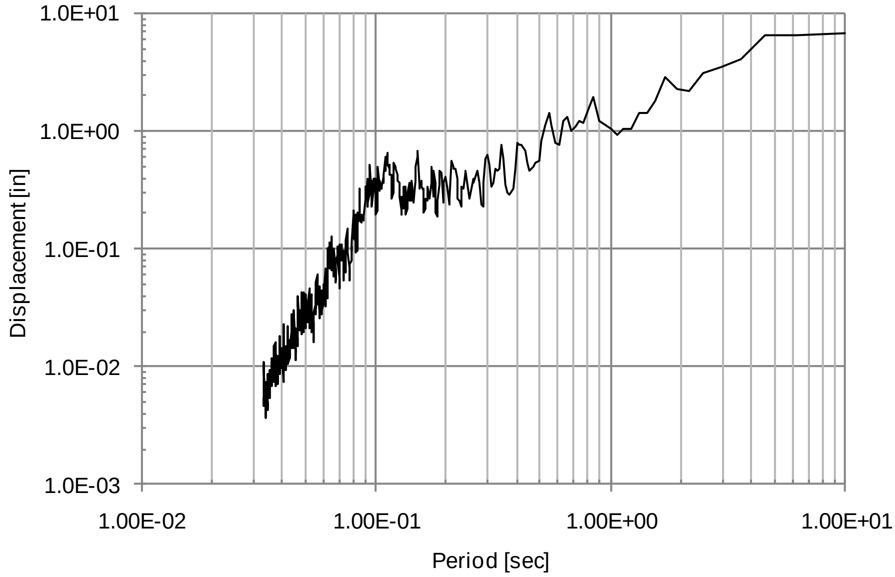


Table 4.11.1 Maximum displacement and velocity at the top of the column provided by transient analysis

Result type		Displacement [*mm*]	Velocity [*m/sec*]
Reference		*59.2*	*0.508*
Analysis type	Number of elements		
Direct transient	10	*58.9*	*0.411*
	20	*58.9*	*0.410*
	50	*58.9*	*0.410*
Modal transient	10	*59.2*	*0.433*
	20	*59.1*	*0.434*
	50	*59.1*	*0.435*

Figure 4.11.4
Velocity spectra for the period range 0.03~10 sec

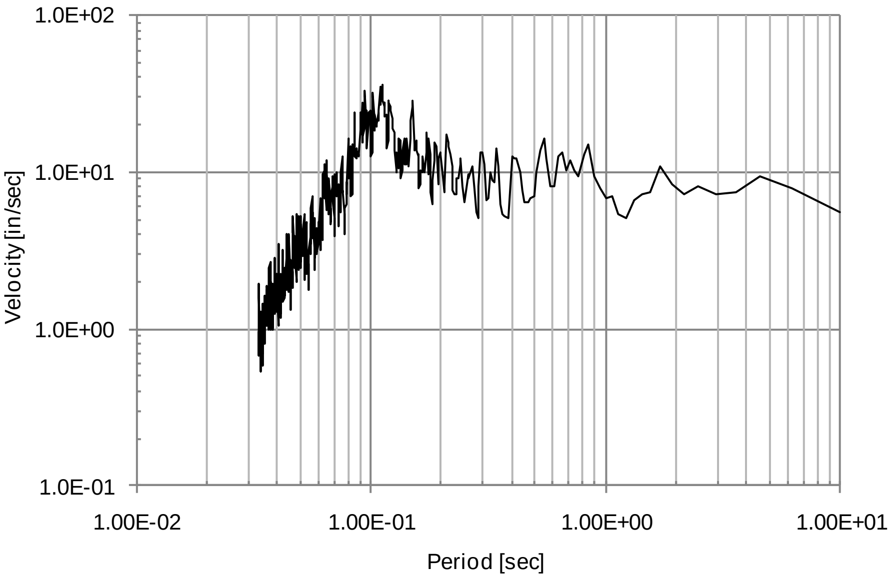


Table 4.11.2 Maximum displacement and velocity at the top of the column provided by response spectrum analysis

Result type			Displacement [*mm*]	Velocity [*m/sec*]
Reference			*59.2*	*0.508*
Number of elements	Spectrum type	Combination method		
10	Displacement	ABS	*67.2*	*0.639*
		SRSS	*57.0*	*0.392*
	Velocity	ABS	*70.8*	*0.640*
		SRSS	*61.0*	*0.395*
20	Displacement	ABS	*67.2*	*0.638*
		SRSS	*57.0*	*0.392*
	Velocity	ABS	*70.8*	*0.639*
		SRSS	*61.0*	*0.395*
50	Displacement	ABS	*67.2*	*0.639*
		SRSS	*57.0*	*0.392*
	Velocity	ABS	*70.8*	*0.639*
		SRSS	*61.0*	*0.395*

4.12 Rayleigh Wave Velocity

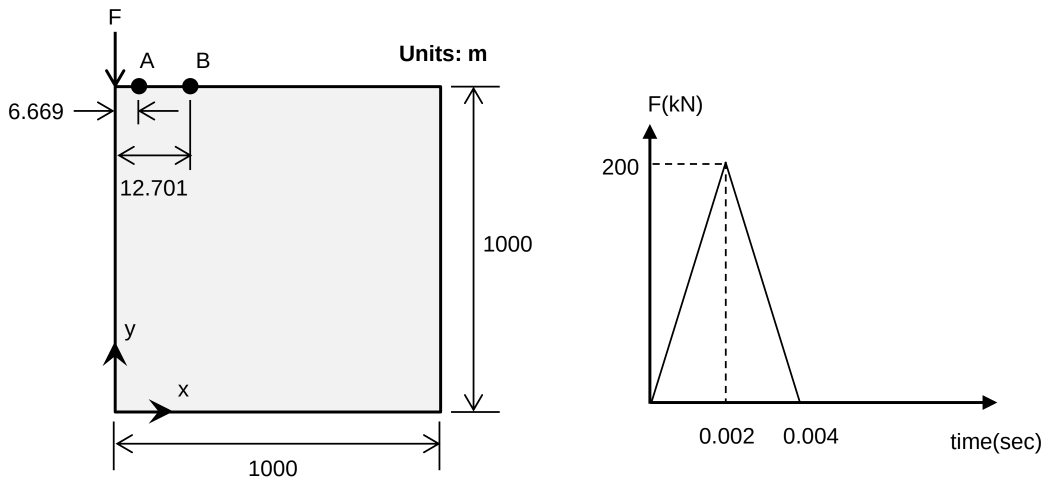
REFERENCE	Knopoff, L.[8,9]
ELEMENTS	Axisymmetric elements
MODEL FILENAME	Dynamics12.gts

Figure 4.12.1 shows an axisymmetric model setup for the measurement of the Rayleigh wave velocity. Concentrated load is applied on the top surface in the form of triangular pulse. Direct and modal linear time history analyses are carried out to measure the peak time of Rayleigh wave propagation at point A and B. The time lag and the distance between A and B are used to compute the wave velocity. The calculated Rayleigh wave velocities are compared with theoretical value:

$$V_R = 0.54\sqrt{\frac{(1-\nu)Eg}{(1+\nu)(1-2\nu)\gamma}}$$

where *E* is the Young's modulus, ν is the Poisson's ratio, γ is weight density and g is the gravitational acceleration.

Figure 4.12.1
Axisymmetric soil model and pulse profile



Material data	Young's modulus	*E = 750000 N/m²*
	Poisson's ratio	ν = 0.3
	Unit Weight	γ *= 20000 N/m³*
Section property	Thickness	*t = 1.0 m*

Figure4.12.2
Vertical displacements at point A and B in direct time history

Figure 4.12.2
Vertical displacements vs. time at point A and B obtained using direct time history analysis

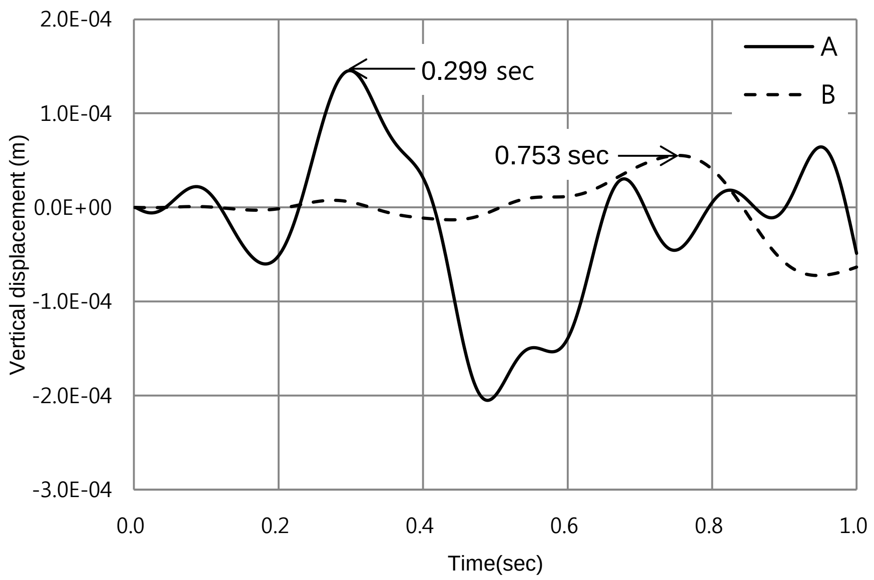


Figure 4.12.3
Vertical displacements vs. time at point A and B obtained using modal time history analysis

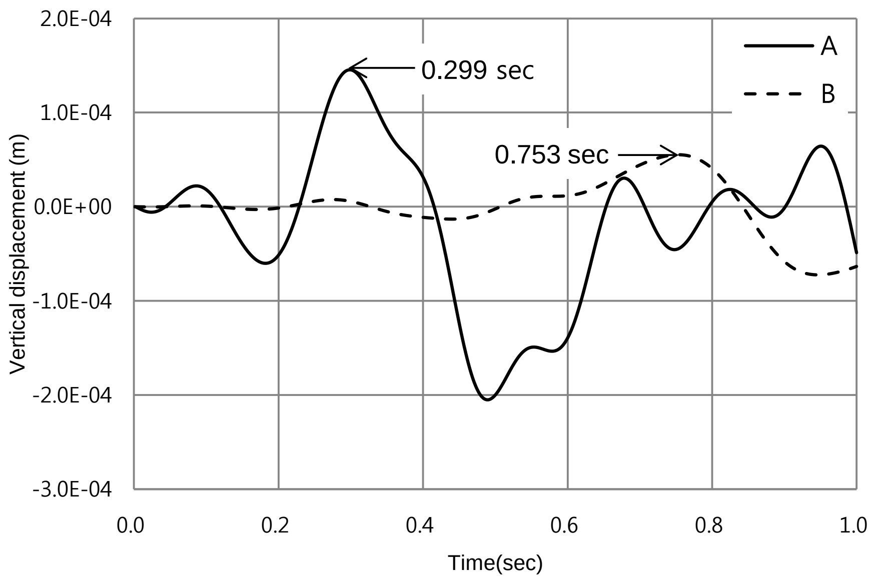


Table 4.12.1 Rayleigh wave velocity between AB

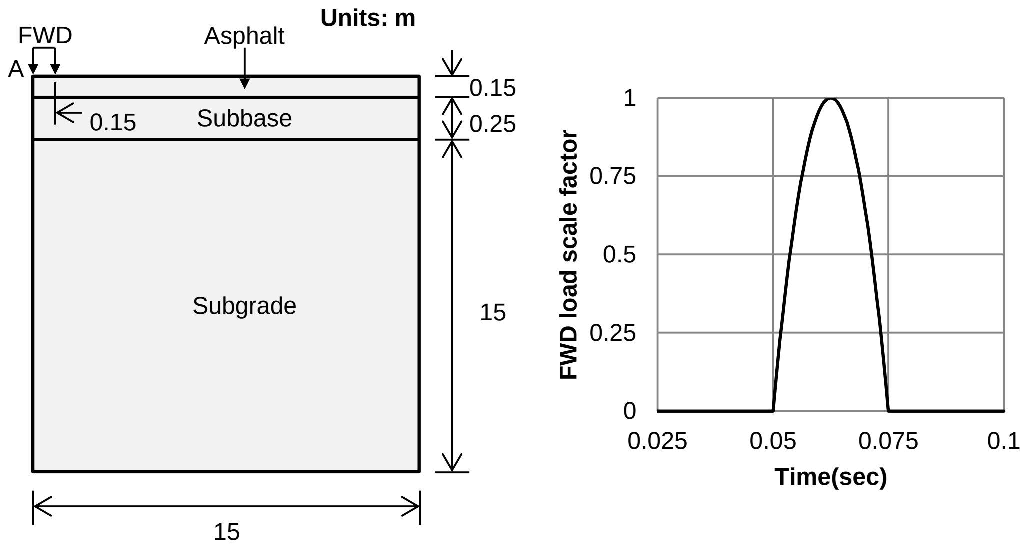
Result type	Velocity [*m/sec*]
Reference	*12.01*
Direct Time History	*13.28*
Modal Time History	*13.28*

4.13 FWD Load on Pavement

REFERENCE	Al-Khoury, R. et al.[10]
ELEMENTS	Axisymmetric element
MODEL FILENAME	Dynamics13.gts

Figure 4.13.1 shows the axisymmetric model to obtain the transient response of layered pavement subjected to FWD (falling weight deflectormeter) load pulse. The pavement is a three layered media containing asphalt, subbase and subgrade. The magnitude of FWD is *50kN* and the radius of FWD is *0.15m* from the axis of symmetry. Direct and modal linear time history analyses are carried out using axisymmetric elements to obtain the vertical displacement at point A. A semi-analytical solution computed by the spectral element method by adopted for comparison.

Figure 4.13.1
Axisymmetric model of layered pavement and FWD load pulse profile



Material data 1 (Asphalt)	Young's modulus Poisson's ratio Unit Weight	*E = 1000 MPa* ν = *0.35* γ = *2.3 kN/m^3*
Material data 2 (Subbase)	Young's modulus Poisson's ratio Unit Weight	*E = 200 MPa* ν = *0.3* γ = *2.0 kN/m^3*
Material data 3 (Subgrade)	Young's modulus Poisson's ratio Unit Weight	*E = 100 MPa* ν = *0.3* γ = *1.5 kN/m^3*
Section property	Thickness	*t = 1.0 m*

Figure 4.13.2
Vertical displacement at A vs. time obtained using direct and modal linear time history analyses

Figure 4.13.2
Vertical displacement at A vs. time obtained using direct and modal linear time history analyses

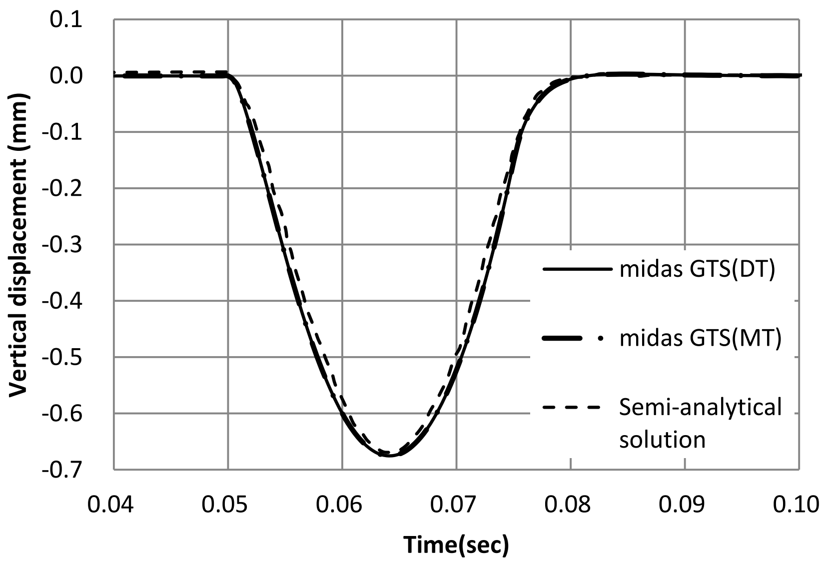


Table 5.1.1 Maximum vertical displacement at the center of symmetry

Result type	Vertical displacement [*m/sec*]
Reference	*0.669*
Direct Time History	*0.675*
Modal Time History	*0.675*

4.14 Earthquake Response of a Layered Soil Deposit

REFERENCE	Schnabel, P. B. et al.[11]
KEYWORDS	free field, equivalent linear
MODEL FILENAME	Dynamic14.lif, Dynamic14.gts

Figure 4.14 shows a 150-ft soil profile consisting of clay and sand overlying a half-space. The material parameters of individual layers are summarized in Tables 4.14.1 ~ 4.14.4. The soil deposit is subjected to controlled motion corresponding to the earthquake time history which had been recorded at Diamond Heights (EW component) during the Loma Prieta earthquake. This motion is normalized to a peak acceleration of 0.1g. Free-field and equivalent linear analyses are carried out to obtain various responses of the soil deposit. The obtained responses are compared with the results included in the reference.

Figure 4.14.1
Soil profile

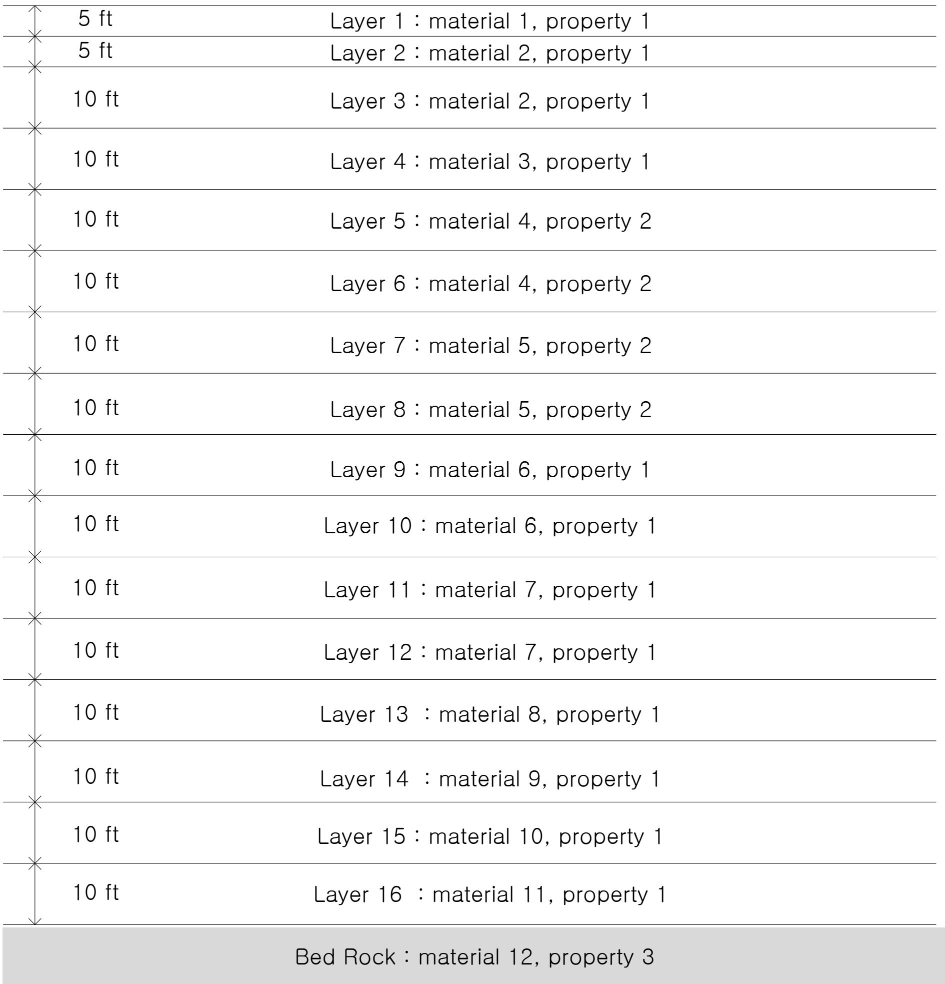


Table 4.14.1 Material data of soil profile

Material data	Unit Weight (kgf / ft^3)	Maximum shear modulus (kgf / ft^2)	Damping ratio
1	0.125	3882.0	0.05
2	0.125	3144.0	0.05
3	0.125	3503.0	0.05
4	0.125	3882.0	0.05
5	0.125	4697.0	0.05
6	0.130	6823.0	0.05
7	0.130	7913.0	0.05
8	0.130	9084.0	0.05
9	0.130	10335.0	0.05
10	0.130	13081.0	0.05
11	0.140	69565.0	0.01

Table 4.14.2 Strain compatible property 1(sand)

Sand (Seed and Idriss 1970)		Sand (Idriss 1990)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.000	1.0×10^{-6}	0.0024
3.0×10^{-6}	1.000	3.0×10^{-6}	0.0042
1.0×10^{-5}	0.990	1.0×10^{-5}	0.0080
3.0×10^{-5}	0.960	3.0×10^{-5}	0.0140
1.0×10^{-4}	0.850	1.0×10^{-4}	0.0280
3.0×10^{-4}	0.640	3.0×10^{-4}	0.0510
1.0×10^{-3}	0.370	1.0×10^{-3}	0.0980
3.0×10^{-3}	0.180	3.0×10^{-3}	0.1550
1.0×10^{-2}	0.080	1.0×10^{-2}	0.2100
3.0×10^{-2}	0.050	3.0×10^{-2}	0.2500
1.0×10^{-1}	0.035	1.0×10^{-1}	0.2800

Table 4.14.3 Strain compatible property 2(clay)

Clay (Seed and Sun 1989)		Clay (Idriss 1990)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.000	1.00×10^{-6}	0.0024
3.0×10^{-6}	1.000	3.00×10^{-6}	0.0042
1.0×10^{-5}	1.000	1.00×10^{-5}	0.0080
3.0×10^{-5}	0.981	3.00×10^{-5}	0.0140
1.0×10^{-4}	0.941	1.00×10^{-4}	0.0280
3.0×10^{-4}	0.847	3.00×10^{-4}	0.0510
1.0×10^{-3}	0.656	1.00×10^{-3}	0.0980
3.0×10^{-3}	0.438	3.00×10^{-3}	0.1550
1.0×10^{-2}	0.238	1.00×10^{-2}	0.2100
3.0×10^{-2}	0.144	3.16×10^{-2}	0.2500
1.0×10^{-1}	0.110	1.00×10^{-1}	0.2800

Table 4.14.4 Strain compatible property 3(rock)

Rock (Idriss)		Rock (Idriss)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.0000	1.00×10^{-6}	0.004
3.0×10^{-6}	1.0000	1.00×10^{-5}	0.008
1.0×10^{-5}	0.9875	1.00×10^{-4}	0.015
3.0×10^{-5}	0.9525	1.00×10^{-3}	0.030
1.0×10^{-4}	0.9000	1.00×10^{-2}	0.046
3.0×10^{-4}	0.8100		
1.0×10^{-3}	0.7250		
1.0×10^{-2}	0.5500		

Figure 4.14.2
Controlled acceleration history corresponding to Diamond Heights earthquake (EW components)

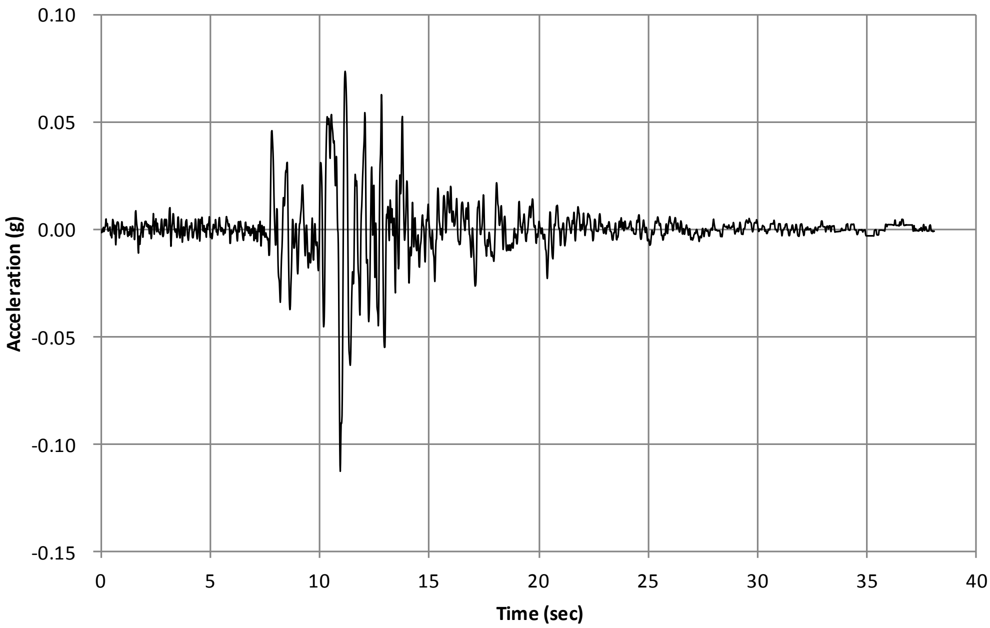


Table 4.14.5 Maximum acceleration with depth

	Free Field	2D Equivalent	Reference
Depth (ft)	Maximum Acceleration(g)	Maximum Acceleration(g)	Maximum Acceleration(g)
0	*0.19033*	*0.19081*	*0.19037*
5	*0.18999*	*0.19035*	*0.19006*
10	*0.18866*	*0.18875*	*0.18876*
20	*0.18253*	*0.18275*	*0.18258*
30	*0.17206*	*0.17248*	*0.17208*
40	*0.15946*	*0.16018*	*0.15947*
50	*0.14289*	*0.14387*	*0.14288*
60	*0.12653*	*0.12742*	*0.12652*
70	*0.11042*	*0.11022*	*0.11050*
80	*0.09837*	*0.09825*	*0.09840*
90	*0.08997*	*0.08955*	*0.08999*
100	*0.08266*	*0.08216*	*0.08268*
110	*0.08562*	*0.08586*	*0.08559*
120	*0.08548*	*0.08568*	*0.08547*
130	*0.08201*	*0.08216*	*0.08198*
140	*0.07767*	*0.07750*	*0.07769*
150 (Within)	*0.07614*	*0.07613*	*0.07617*
150 (Outcrop)	*0.09999*	-	*0.10000*

Figure 4.14.3
Acceleration time history at ground surface obtained using free field and equivalent linear analyses

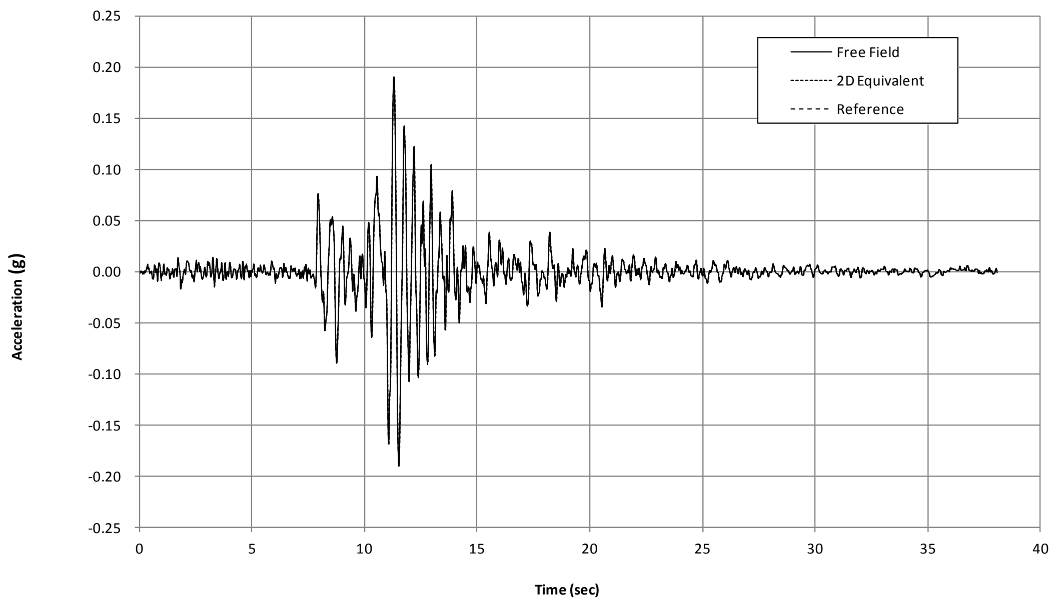


Table 4.14.6 Maximum strain and stress with depth

	Free Field		2D Equivalent		Reference	
Depth (ft)	Maximum strain (%)	Maximum stress (lb/ft^2)	Maximum strain (%)	Maximum stress (lb/ft^2)	Maximum strain (%)	Maximum stress (lb/ft^2)
2.5	*0.00154*	*59.42*	*0.00153*	*59.01*	*0.00154*	*59.43*
7.5	*0.00591*	*178.39*	*0.00587*	*177.33*	*0.00591*	*178.41*
15.0	*0.01267*	*355.30*	*0.01262*	*353.47*	*0.01267*	*355.34*
25.0	*0.01952*	*582.73*	*0.01944*	*580.05*	*0.01952*	*582.77*
35.0	*0.02198*	*795.86*	*0.02187*	*791.83*	*0.02198*	*795.90*
45.0	*0.02806*	*993.48*	*0.02792*	*988.11*	*0.02806*	*993.53*
55.0	*0.02723*	*1169.80*	*0.02708*	*1162.92*	*0.02723*	*1169.91*
65.0	*0.03132*	*1327.90*	*0.03112*	*1318.60*	*0.03132*	*1328.02*
75.0	*0.02711*	*1464.70*	*0.02680*	*1447.55*	*0.02711*	*1464.83*
85.0	*0.03010*	*1585.30*	*0.02959*	*1559.73*	*0.03011*	*1585.55*
95.0	*0.02671*	*1679.60*	*0.02622*	*1650.40*	*0.02671*	*1679.89*
105.0	*0.02825*	*1752.40*	*0.02772*	*1722.40*	*0.02826*	*1752.80*
115.0	*0.02465*	*1814.10*	*0.02439*	*1795.89*	*0.02466*	*1814.66*
125.0	*0.02561*	*1867.70*	*0.02534*	*1849.19*	*0.02563*	*1868.38*
135.0	*0.02229*	*1910.20*	*0.02211*	*1895.24*	*0.02230*	*1910.82*
145.0	*0.01729*	*1952.50*	*0.01709*	*1930.27*	*0.01729*	*1952.63*

Figure 4.14.4
Absolute acceleration response spectra of layer 1 for damping ratio of 5%

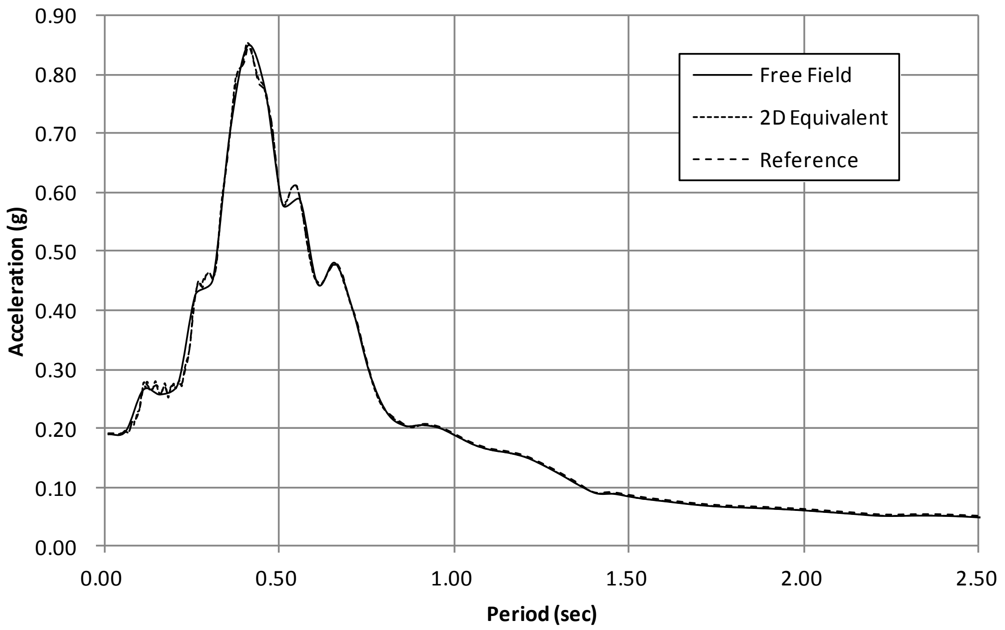
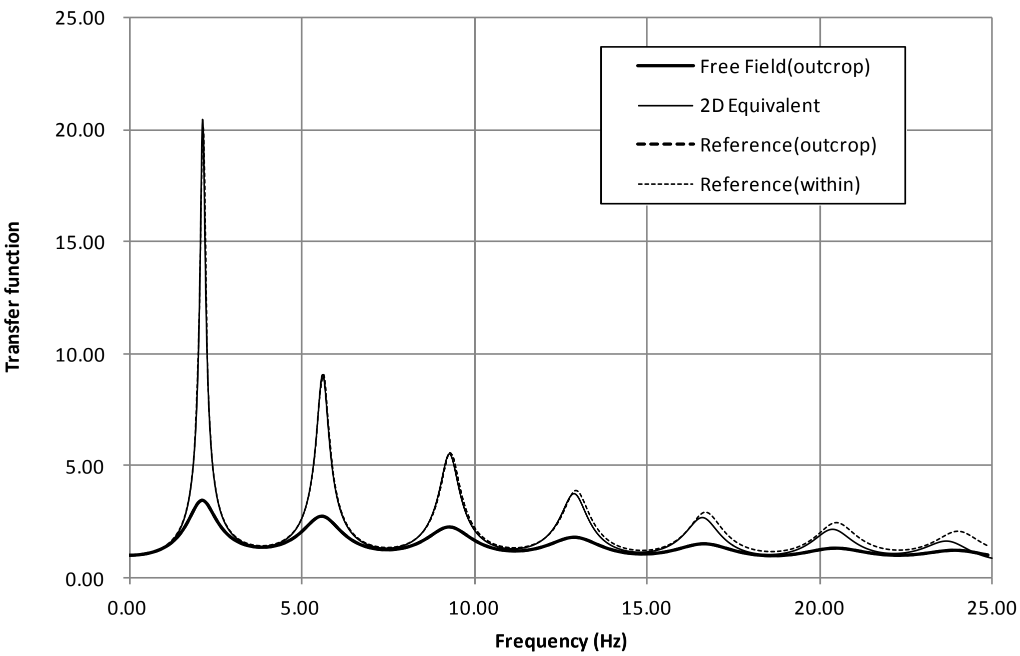


Figure 4.14.5
Transfer function of acceleration between layer 1 and layer 17

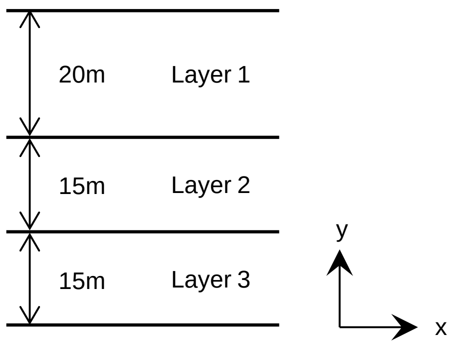


4.15 Earthquake Response of a Layered Soil Deposit

REFERENCE	Nielsen, A. H.[12] and Zienkiewicz et al.[13]
KEYWORDS	Free field element
MODEL FILENAME	Dynamic15.gts

Figure 4.15.1 shows a flat site with horizontal soil layers. The flat site is subjected to enforced acceleration at the bottom. Direct linear transient analyses are carried out for two different modeling cases. In Case 1, the left and right edges of the model are constrained to have identical displacements utilizing multi-point constraints. This gives reference free field response for the infinite flat site. In Case 2, the horizontal soil layers are modeled using free field elements placed at the lateral boundaries. These models should give identical free field responses. Figure 4.15.2 shows the applied acceleration history at the bottom of the model. The same enforced motion is applied for both x and y directions. The acceleration histories at the top surface of both models are compared.

Figure 4.15.1
Flat site profile



Material data 1 (Layer 1)	Young's modulus	$E = 300\ MPa$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 19.6\ kN/m^3$
Material data 2 (Layer 2)	Young's modulus	$E = 700\ MPa$
	Poisson's ratio	$\nu = 0.4$
	Unit weight	$\gamma = 23.52\ kN/m^3$
Material data 3 (Layer 3)	Young's modulus	$E = 2000\ MPa$
	Poisson's ratio	$\nu = 0.25$
	Unit weight	$\gamma = 26.46\ kN/m^3$
Section	Free field width factor	10^5

Figure 4.15.2
Applied ground acceleration profile

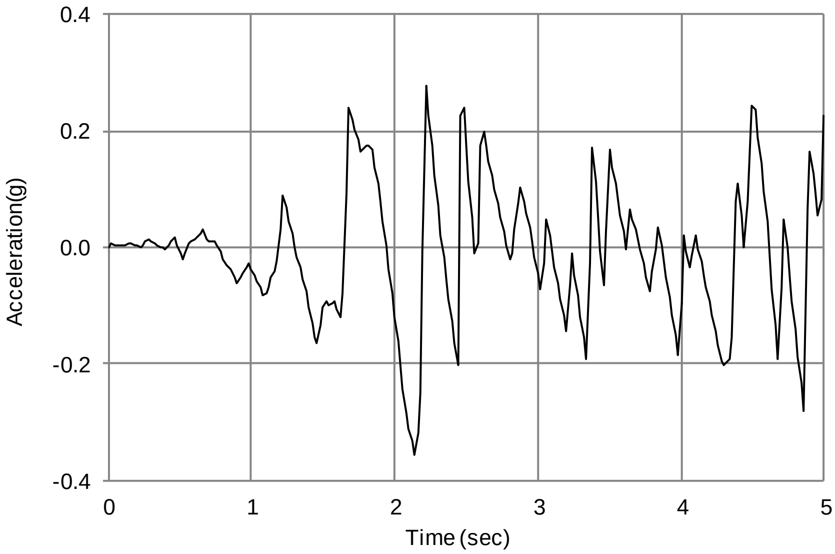


Figure 4.15.3
Horizontal acceleration history at the top surface of the model obtained using free-field elements and tied boundary conditions

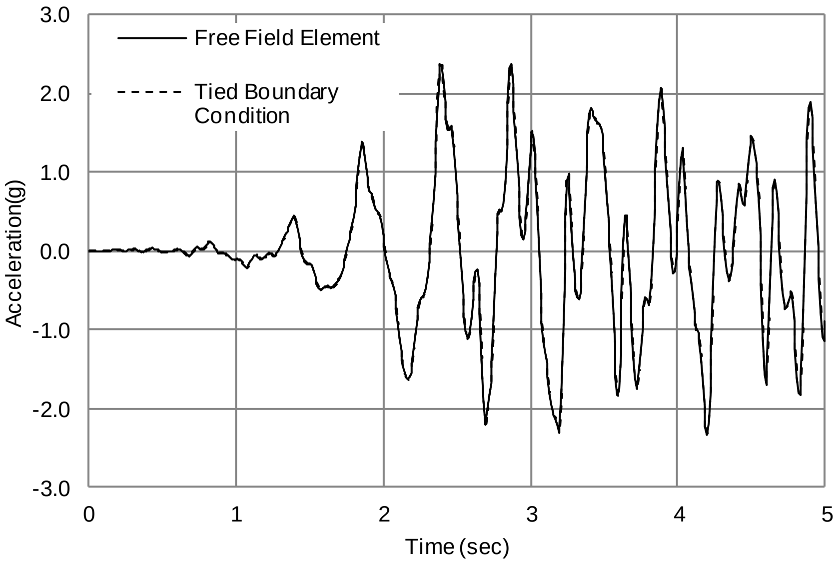
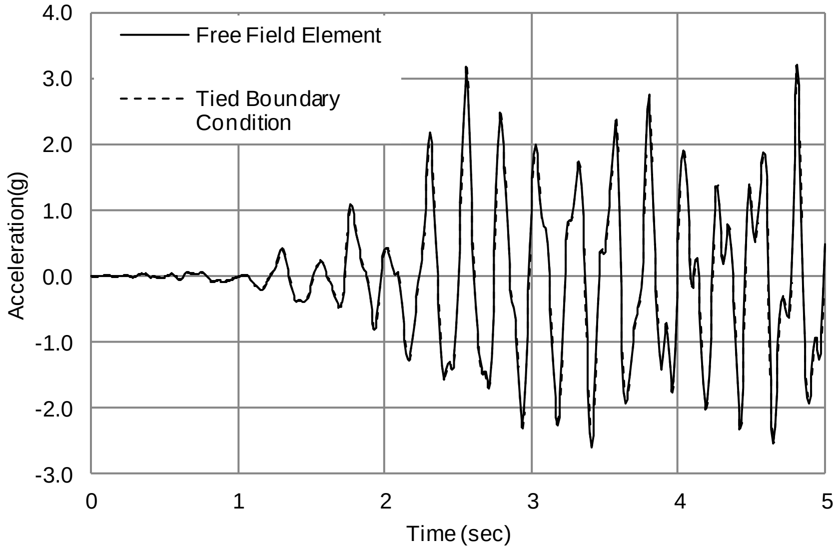


Figure 4.15.4
Vertical acceleration history at the top surface of the model obtained using free-field elements and tied boundary conditions



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