5.1 Unconfined Flow on a Horizontal Impermeable Boundary

REFERENCE	Harr, M. E.[1]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation01.gts

In this example an unconfined two-dimensional ground water flow through soil layer is considered. The dimension of soil layer and boundary conditions are described in Figure 5.1.1. Total head of 2 m and 1 m is prescribed on the left and right side of the soil layer, respectively. The soil layer is impermeable at the bottom. The permeability is 1 m/day and van Genuchten model is used for the unsaturated soil. The theoretical solution of this problem is calculated by Dupuit.

The total discharge through AB obtained by steady-state seepage analysis is compared with Dupuit's solution:

$$q = k\frac{h_1^2 - h_2^2}{2L}$$

where h_1 and h_2 are the boundary head values, L is the horizontal length and k is the permeability.

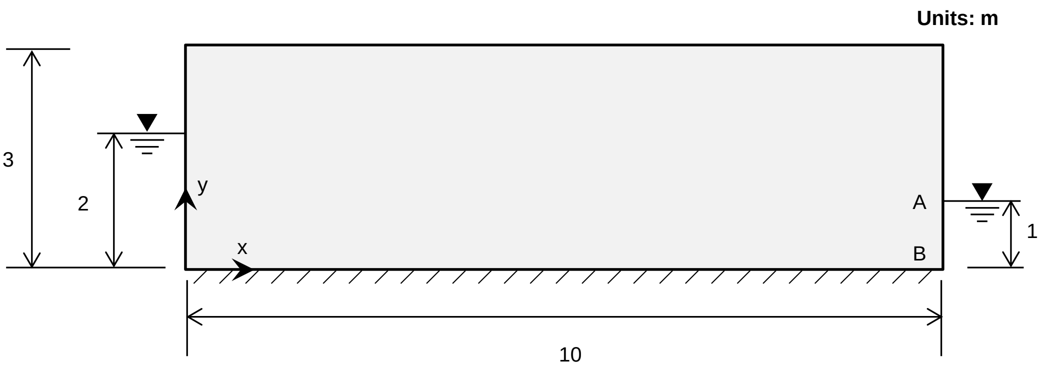


Figure 5.1.1
Soil layer with head boundary condition

Material data	Permeability Van Genuchten	*1.0 m/day* θ_r =0.023, θ_s =0.366 *a = 4.3, n = 1.5206, m = 0.3423*
Section property	Thickness	*t = 1.0 m*

Table 5.1.1 Total discharge in the cross section AB obtained using plane strain elements

		total discharge [m^3/day/m]
Reference		*0.15*
Element type	Number of elements	
TRIA-3	15×(50×2)	*0.1553*
QUAD-4	15x50	*0.1553*

Table 5.1.2 Total discharge in the cross section AB obtained using solid elements

		total discharge [m^3/day/m]
Reference		*0.15*
Element type	Number of elements	
PENTA-6	15×(50×2)x1	*0.1553*
PYRAM-5	15x(50x4)x1	*0.1553*
TETRA-4	15×(50×4)x2	*0.1553*
HEXA-8	15x50x1	*0.1553*

5.2 Flow through an Earth Dam

REFERENCE	Harr, M. E.[1]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation02.gts

Figure 5.2.1 shows ground water seepage problem of a triangular shape dam. Steady-state seepage analysis is carried out with total head of 1 m prescribed on the left side of the dam, designated AC. The right side CB is designated as seepage face i.e., review-boundary condition is applied. The bottom surface of the dam is impermeable. The permeability is 1 m/day and unsaturated property is not considered. The total discharge through CB is compared with theoretical solution:

$$q = \frac{kh}{2}\tan\left(\angle CBA\right)$$

where h is the prescribed boundary total head on AC, and k is the permeability.

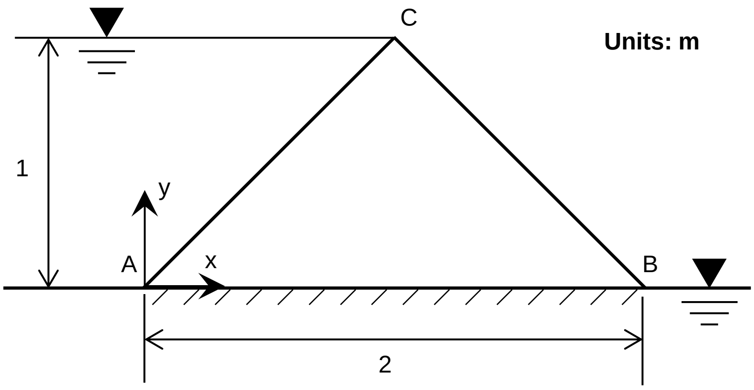


Figure 5.2.1
Earth dam with head boundary condition

Material data	Permeability	*1.0 m/day*
Section property	Thickness	*t = 1.0 m*

Table 5.2.1 Total discharge in the cross section BC obtained using plane strain elements

		total discharge [$m^3/day/m$]
Reference		*0.5*
Element type	Number of elements	
TRIA-3	400	*0.5*

Table 5.2.2 Total discharge in the cross section BC obtained using solid elements

		total discharge [$m^3/day/m$]
Reference		*0.5*
Element type	Number of elements	
PENTA-6	400	*0.5*

5.3 Confined Flow around an Impermeable Wall

REFERENCE	Harr, M. E.[1]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation03.gts

A confined flow problem with an impermeable wall is depicted in Figure 5.3.1. Steady-state flow develops due to the difference in total head in the horizontal direction, affected by the impermeable wall in the middle. The bottom surface of the soil layer is impermeable. The permeability is 1 m/day and unsaturated property is not considered. The total discharge through CB obtained from steady-state seepage analysis is compared with a theoretical solution.

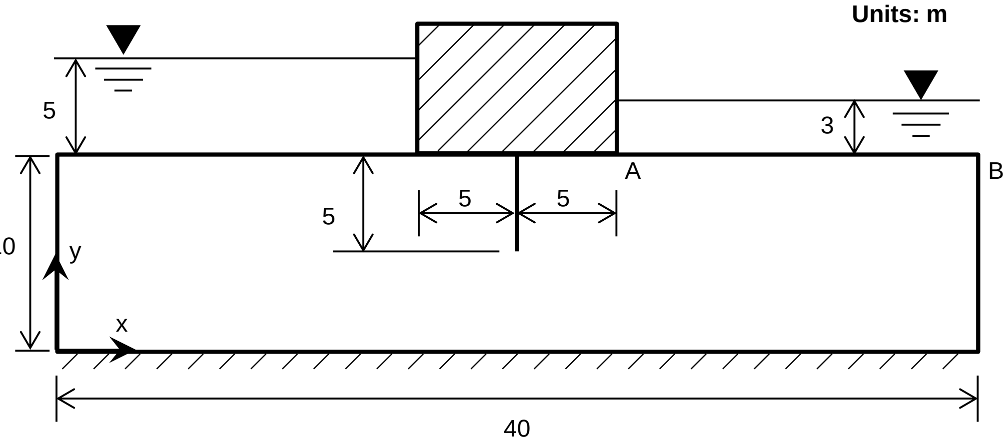


Figure 5.3.1
Soil layer with impermeable wall and dam

Material data	Permeability	*1.0 m/day*
Section property	Thickness	*t = 1.0 m*

Table 5.3.1 Total discharge in the cross section AB obtained using plane strain elements

		total discharge [m^3/*day*/*m*]
Reference		*0.8*
Element type	Number of elements	
TRIA-3	20×(80×2)	*0.83*
QUAD-4	20x80	*0.82*

Table 5.3.2 Total discharge in the cross section AB obtained using solid elements

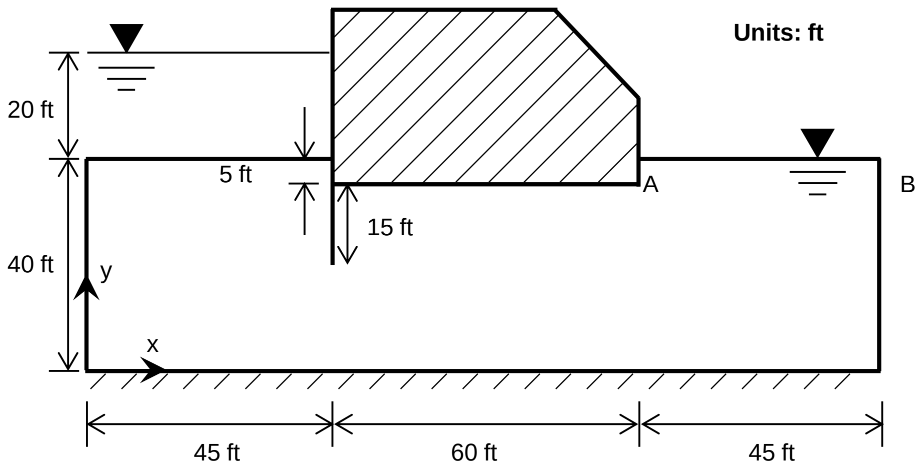
		total discharge [m^3/day/m]
Reference		*0.8*
Element type	Number of elements	
PENTA-6	20×(80×2)x1	*0.831*
PYRAM-5	20x(80x6)x1	*0.824*
TETRA-4	20x(80x11)x1	*0.827*
HEXA-8	20x80x1	*0.822*

5.4 Confined Flow around an Impermeable Dam with a Wall

REFERENCE	Lambe, T.W. et al[2]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation04.gts

A confined flow problem with an impermeable dam and wall is depicted in Figure 5.4.1. Steady-state flow develops due to the difference in total head in the horizontal direction, also affected by the existence of impermeable dam and wall. The bottom surface of the soil layer is impermeable. The permeability is 0.001 ft/day and unsaturated property is not considered. The total discharge through AB obtained from steady-state seepage analysis is compared with a theoretical solution.

Figure 5.4.1
Impermeable dam with a wall and soil layer



Material data	Permeability	*0.001 ft/min*
Section property	Thickness	*t = 1.0 ft*

Table 5.4.1 Total discharge in the cross section AB obtained using plane strain elements

		total discharge [ft^3*/min/ft*]
Reference		5.76×10^{-3}
Element type	Number of elements	
QUAD-4	228	5.72×10^{-3}

Table 5.4.2 Total discharge in the cross section AB obtained using solid elements

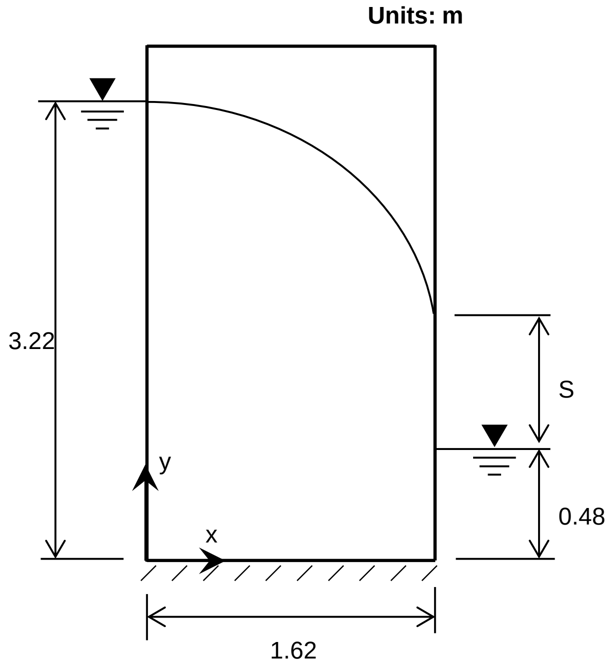
		total discharge [$ft^3/min/ft$]
Reference		5.76×10^{-3}
Element type	Number of elements	
HEXA-8	288	5.72×10^{-3}

5.5 Unconfined Flow through a Dam

REFERENCE	Lee, K. K.[3]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation05.gts

Figure 5.5.1 shows an unconfined flow problem with various boundary conditions. Total head of 3.22m is assigned on the left side and combination of seepage face condition and total head of 0.48m is assigned on the right side. Steady-state ground flow develops with length of seepage face; S. Steady-state seepage analysis is carried out with the permeability of 1 m/day. Van Genuchten model is used for the unsaturated soil. The resulting length of seepage face S is compared with a theoretical solution.

Figure 5.5.1
Dam with head boundary condition and seepage face



Material data	Permeability Van Genuchten	*1.0 m/day* θ_r =0.025, θ_s =0.403 *a = 3.83, n = 1.3774, m = 0.2740*
Section property	Thickness	*t = 1.0 m*

Table 5.5.1 The length of seepage face S obtained using plane strain elements

		seepage face length [*m*]
Reference		*1.54*
Element type	Number of elements	
TRIA-3	50x(15x2)	*1.62*
QUAD-4	50x15	*1.62*

Table 5.5.2 The length of seepage face S obtained using solid elements

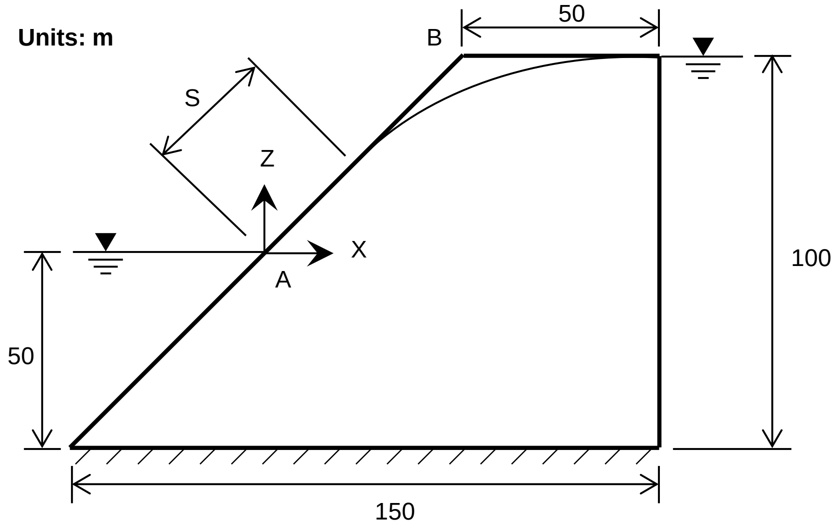
		seepage face length [*m*]
Reference		*1.54*
Element type	Number of elements	
PENTA-6	50x(15x2)x1	*1.62*
PYRAM-5	50x(15x6)x1	*1.62*
TETRA-4	50x(15x20)x1	*1.62*
HEXA-8	50x15x1	*1.62*

5.6 Unconfined Flow toward a Riverbank

REFERENCE	Strack, O. D. et al[4]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation06.gts

Steady-state unconfined ground water flow through a riverbank is evaluated. The dimension of riverbank and boundary conditions are depicted in Figure 5.6.1. Total head of 0 m and 50 m is prescribed on the left and the right side of the riverbank, respectively. Seepage face review boundary condition is assigned on AB. The bottom surface of the soil layer is impermeable. The permeability of the soil is 1.0 x 10^{-6} m/sec and Van Genuchten model is used for the unsaturated soil. The length of seepage face S obtained by steady-state seepage analysis is compared with a theoretical solution.

Figure 5.6.1
Riverbank with head boundary condition and seepage face



Material data	Permeability Van Genuchten	*1.0 x 10^{-6} m/sec* θ_r *=0.023,* θ_s *=0.366* *a = 4.3, n = 1.5206, m = 0.3423*
Section property	Thickness	*t = 1.0 m*

Table 5.6.1 The length of seepage face S obtained using plane strain elements

		seepage face length [*m*]
Reference		*25.5*
Element type	Number of elements	
TRIA-3	60x(20x2)	*25.9*
QUAD-4	60x20	*23.6*

Table 5.6.2 The length of seepage face S obtained using solid elements

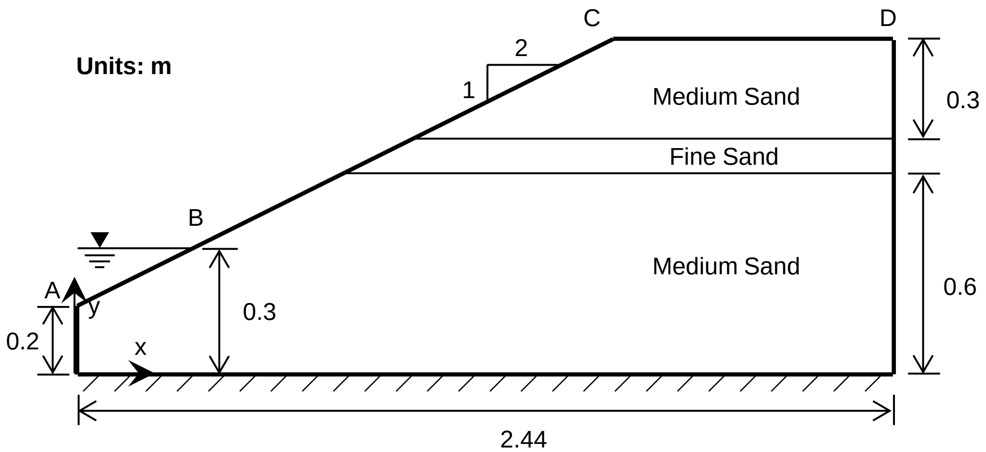
		seepage face length [*m*]
Reference		*25.5*
Element type	Number of elements	
PENTA-6	60x(20x2)x1	*25.9*
TETRA-4	60x(20x6)x1	*23.6*
HEXA-8	60x20x1	*23.6*

5.7 Unconfined Flow in Layered Slope with Multiple Seepage Faces

REFERENCE	Rulon, J. J. et al[5]
ELEMENTS	Plane strain elements, solid elements
MODEL FILENAME	SeepageAndConsolidation07.gts

Figure 5.7.1 shows a layered slope model for steady-state seepage analysis. Surface flux of 2.1 x 10^{-4} m/sec is applied on the upper surface; CD. Due to difference in the permeability of the layers, the resulting flow develops two separate seepage faces. Along the slope, total head of 0.3m is assigned on surface AB and seepage face review boundary condition is assigned on the remaining; surface BC. Analysis is carried out to obtain the total discharge through AC. A theoretical solution is taken for comparison.

Figure 5.7.1
Layered soil and head boundary condition



Material data (Medium Sand)	Permeability	*1.4 x 10^{-3} m/sec*
Material data (Fine Sand)	Permeability	*5.5 x 10^{-5} m/sec*
Section property	Thickness	*t = 1.0 m*

Table 5.7.1 Total discharge in the cross section AC obtained using plane strain elements

		total discharge [$m^3/sec/m$]
Reference		*1.764 x 10*$^{-4}$
Element type	Number of elements	
TRIA-3	1880	*1.764 x 10*$^{-4}$
QUAD-4	940	*1.764 x 10*$^{-4}$

Table 5.7.2 Total discharge in the cross section AC obtained using solid elements

		total discharge [$m^3/sec/m$]
Reference		*1.764 x 10*$^{-4}$
Element type	Number of elements	
PENTA-6	1880	*1.764 x 10*$^{-4}$
TETRA-4	5640	*1.764 x 10*$^{-4}$
HEXA-8	940	*1.764 x 10*$^{-4}$

5.8 Pumping a Radial Well

REFERENCE	Freeze, R. A. et al[6]
ELEMENTS	Axisymmetric elements
MODEL FILENAME	SeepageAndConsolidation08.gts

A transient behavior of pumped radial well is evaluated. The dimension of radial well and boundary conditions are described in Figure 5.7.1. An analytical solution is available in terms of the storativity, transmissivity, and pumping rate of aquifer. These values can be converted to the more familiar material parameters using the following relations:

$$S = m_w \gamma_w b = 0.01$$
$$T = k\,b = 0.01 m^2 / \sec$$

S : Storativity
m_w : Slope of the storage curve
γ_w : Weight density of water
b : Thickness of acquifer
T : Transmissivity
k : Permeability

Transient seepage analysis is carried out with the pumping rate of $Q = 0.125 m^3 / \sec$. The total head at point A and B at different elapsed times are compared with the theoretical solution.

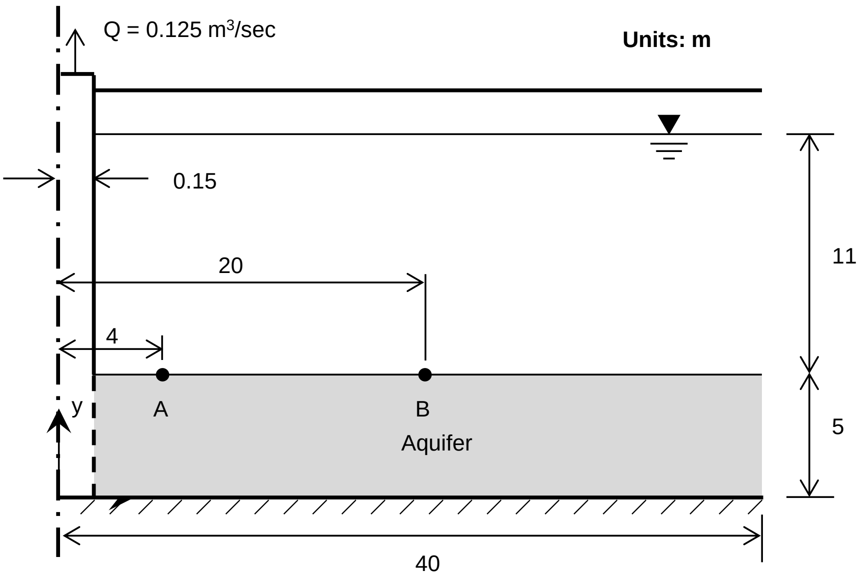


Figure 5.8.1
Radial well geometry and water level

Material data	Permeability	*0.002 m/sec*
	Specific storage	*0.01 m^{-1}*

Slope of storage curve	*1.01972e-06 m^2/N*

Table 5.8.1 Total head at A obtained using axisymmetric elements

Elapsed time [sec]	*total head* [*m*] at A		*total head* [*m*] at B	
	Reference	*QUADX-4*	*Reference*	*QUADX-4*
10	*15.90*	*15.91*	*16.00*	*16.00*
30	*15.70*	*15.62*	*16.00*	*16.00*
70	*15.10*	*15.14*	*16.00*	*16.00*
150	*14.60*	*14.56*	*16.00*	*15.98*
310	*13.90*	*13.92*	*15.90*	*15.90*
630	*13.10*	*13.26*	*15.70*	*15.70*
1270	*12.50*	*12.60*	*15.30*	*15.37*
2170	*11.90*	*12.12*	*14.90*	*15.08*
3970	*11.30*	*11.67*	*14.30*	*14.79*

5.9 Terzaghi Consolidation

REFERENCE	Bowles, J. E.[7]
ELEMENTS	Solid elements, plane strain elements, axisymmetric elements
MODEL FILENAME	SeepageAndConsolidation09_1~3.gts

One-dimensional Terzaghi consolidation problem is depicted in Figure 5.9.1, Pressure of 1kN/m^2 is applied on the upper surface and sustained for the remainder of the consolidation analysis. The draining condition is applied on the upper side while other sides are undrained to allow excess pore pressure to build up and dissipate during the course of the analysis.

Consolidation analysis is carried out to obtain the transient response during period of 100 days using various solid, plane strain and axisymmetric consolidation elements. Distribution of normalized excess pore pressure obtained using plane strain Quad4 elements is plotted along the vertical direction of the model for t=0.02, 5, 30, 50, 100 days (Figure 5.9.2). Normalized excess pore pressure obtained using various solid, plane strain and axisymmetric elements at y=-10m with respect to time are shown in Figure 5.9.3 - 5.9.5. The results are compared with analytical solution given below.

$$\frac{p(y,t)}{q} = \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{M\,y}{H}\right) \exp\left(-\frac{M^2 E\,k\,t}{\gamma_f H^2}\right)$$

$$M = \frac{\pi(2m+1)}{2}$$

Material data	Young's modulus	$E = 100$ *kPa*
	Poisson's ratio	$\nu = 0.0$
	Unit weight of water	$\gamma_w = 9.806$ *kN/m*^{3}
	Permeability	$k = 0.1$ *m/day*

Figure 5.9.1
Terzaghi one-dimensional consolidation problem

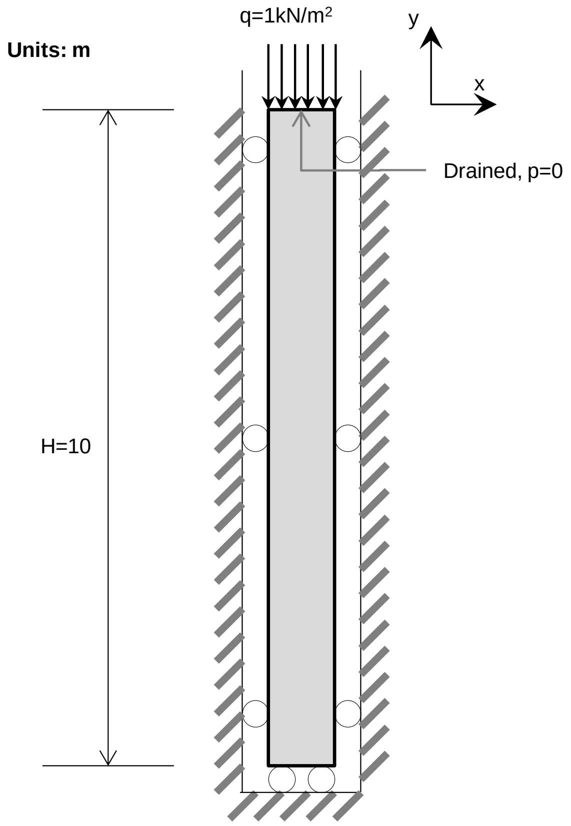


Figure 5.9.2
Normalized excess pore pressure distribution compared with analytical solution

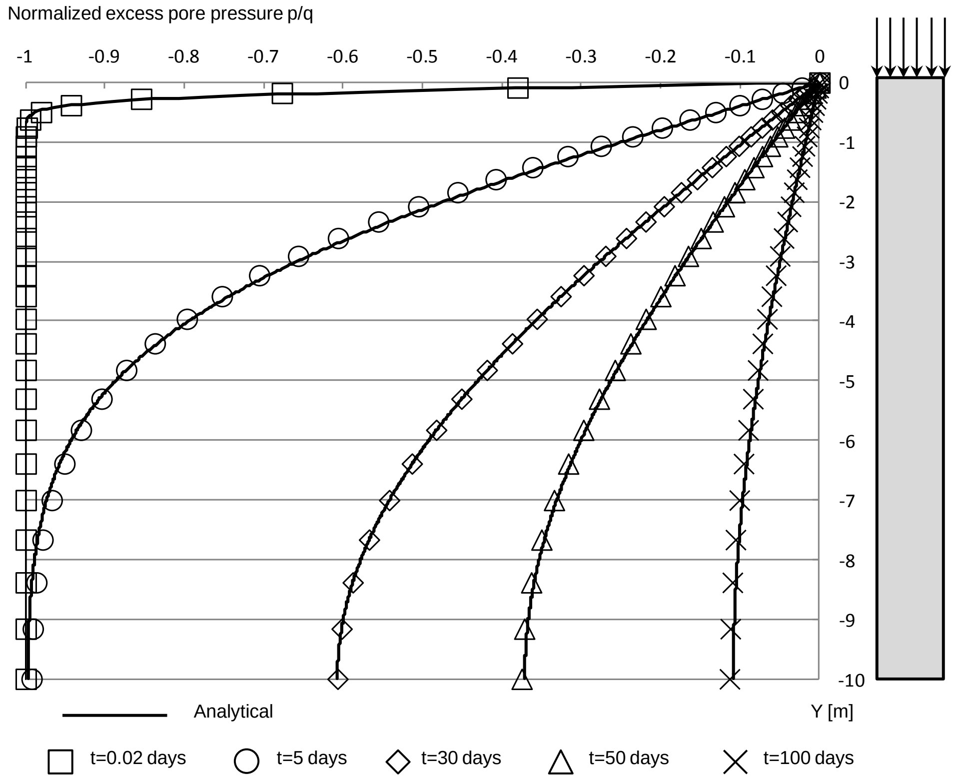


Figure 5.9.3
Dissipation of excess pore pressure with time obtained using solid elements: normalized excess pore pressure at y=-10m compared with analytical solution

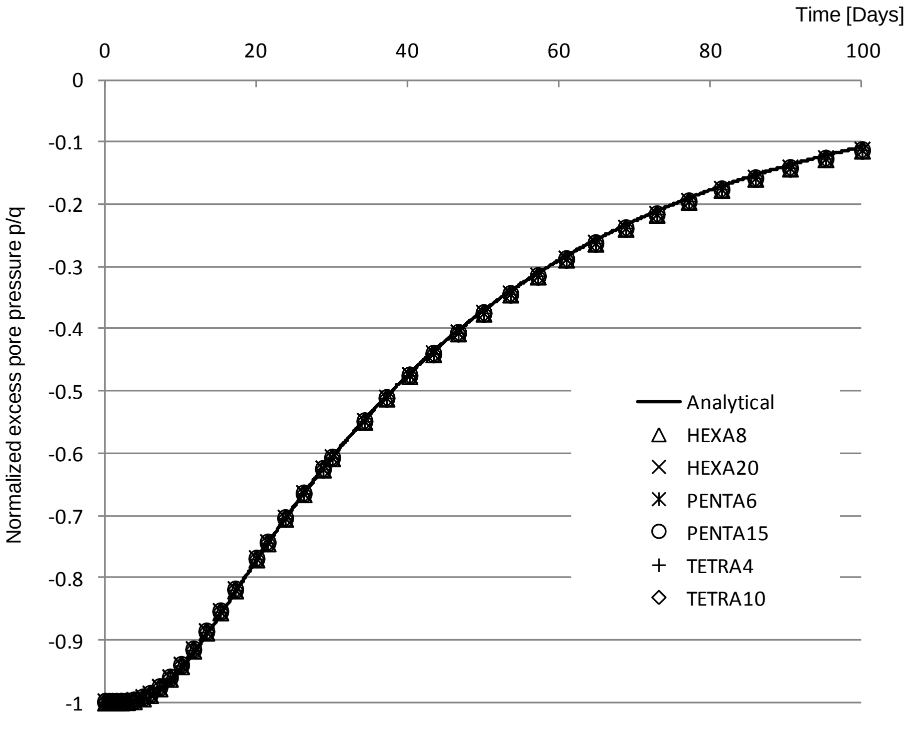


Figure 5.9.4
Dissipation of excess pore pressure with time obtained using plane strain elements: normalized excess pore pressure at y=-10m compared with analytical solution

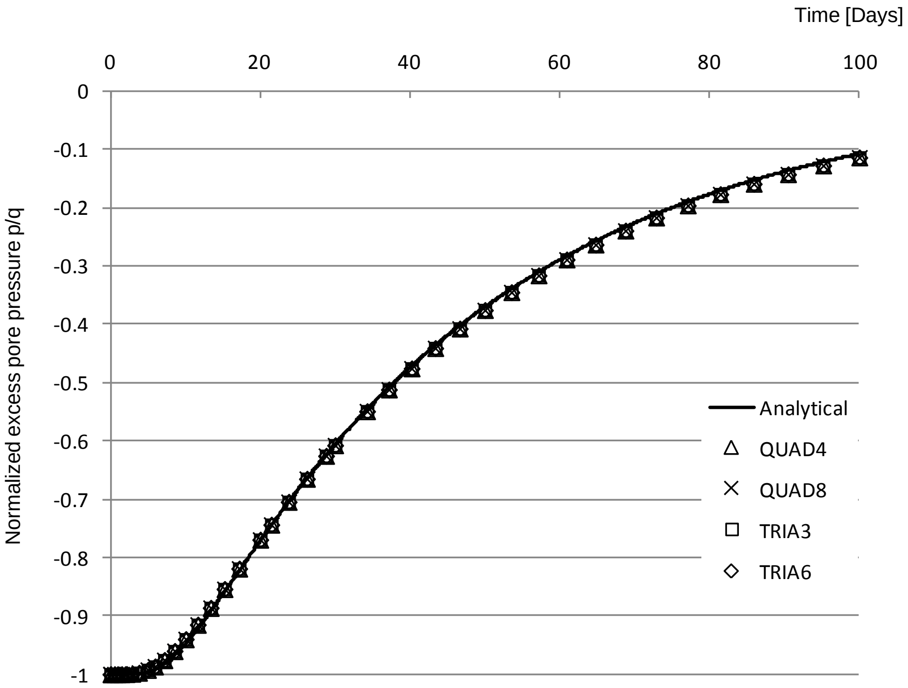
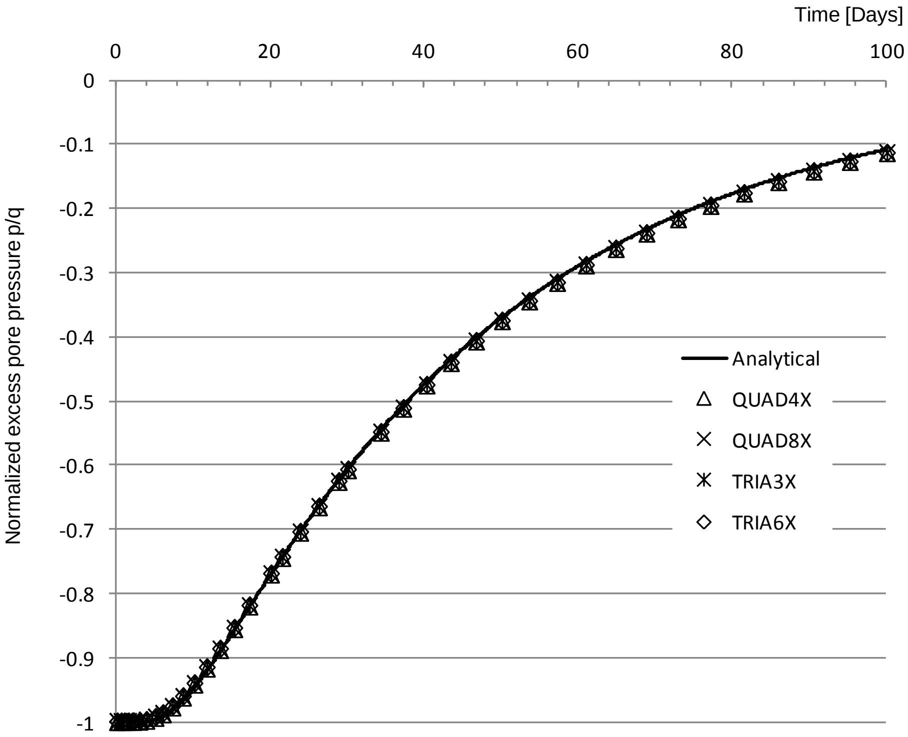


Figure 5.9.5
Dissipation of excess pore pressure with time obtained using axisymmetric elements: normalized excess pore pressure at y=-10m compared with analytical solution



5.10 Stress in Saturated Soil with Seepage

REFERENCE	Das, B. M.[8]
ELEMENTS	Solid elements, plane strain elements, axisymmetric elements
MODEL FILENAME	SeepageAndConsolidation10_1~3.gts

One-dimensional problem with upward seepage is shown in Figure 5.10.1. The problem is solved utilizing seepage-stress sequential analysis. In the seepage analysis, total head is fixed at 12m at the top surface and constant inflow of 0.3m/day is applied on the bottom surface. From the seepage analysis, upward seepage equivalent to the constant hydraulic gradient of 0.3 is obtained. Figure 5.10.2 shows the vertical distribution of pore pressure obtained from the seepage analysis compared with the hydrostatic pore pressure distribution.

Stress analysis is carried out to obtain the vertical stress distribution utilizing the pore pressure field from the seepage analysis. Vertical stresses obtained along the y-axis are compared with the analytical solutions. Total and effective stresses can be obtained analytically in the following form:

$$\sigma_y(y) = -h_f\gamma_f + (H-y)\gamma_{sat}$$
$$\sigma_y'(y) = -(H-y)\left\{\gamma_{sat} - (1+i)\gamma_f\right\}$$

where $i = 0.3$ is the hydraulic gradient in the vertical direction.

Material data	Young's modulus	$E = 100\ kPa$
	Poisson's ratio	$\nu = 0.3$
Weight densities	Water	$\gamma_w = 9.806\ kN/m^3$
	Saturated soil	$\gamma_{sat} = 25\ kN/m^3$
	Unsaturated soil	$\gamma_{unsat} = 17\ kN/m^3$

Figure 5.10.1
Saturated soil model
with upward seepage

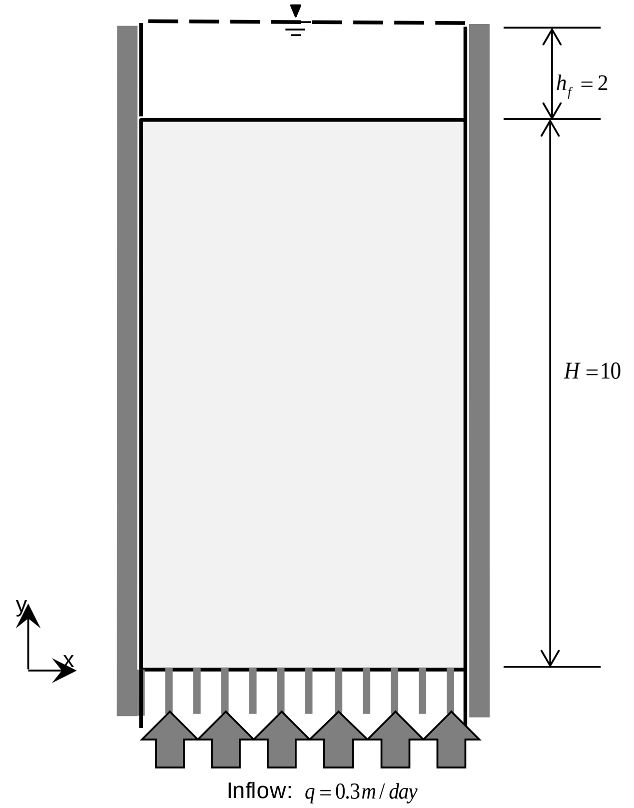


Figure 5.10.2
Pore pressure distribution along y-axis obtained from steady-state seepage analysis compared to hydrostatic pore pressure

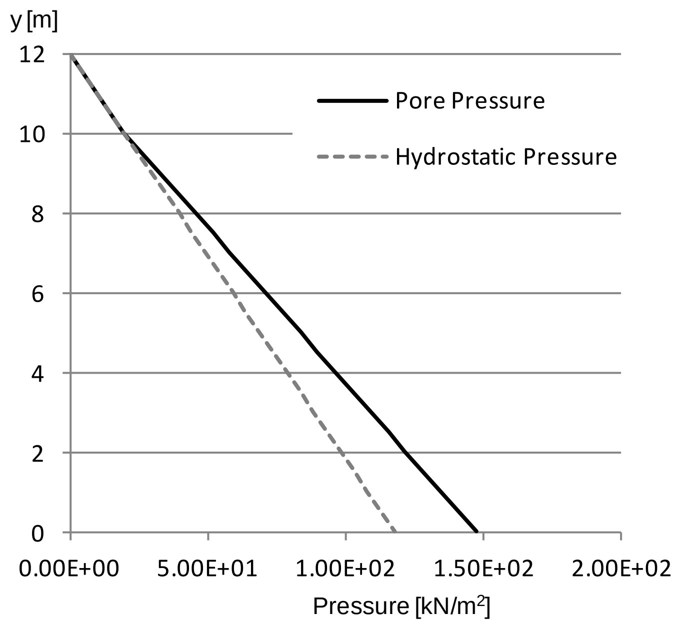


Figure 5.10.2
Total and effective vertical stress distributions in saturated soil obtained using plane strain elements

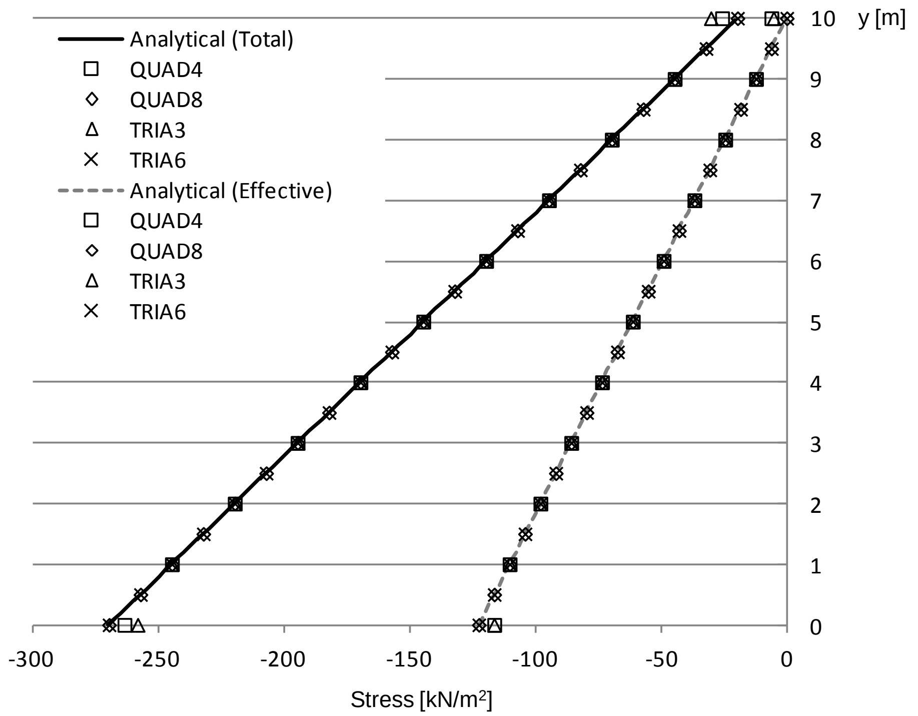
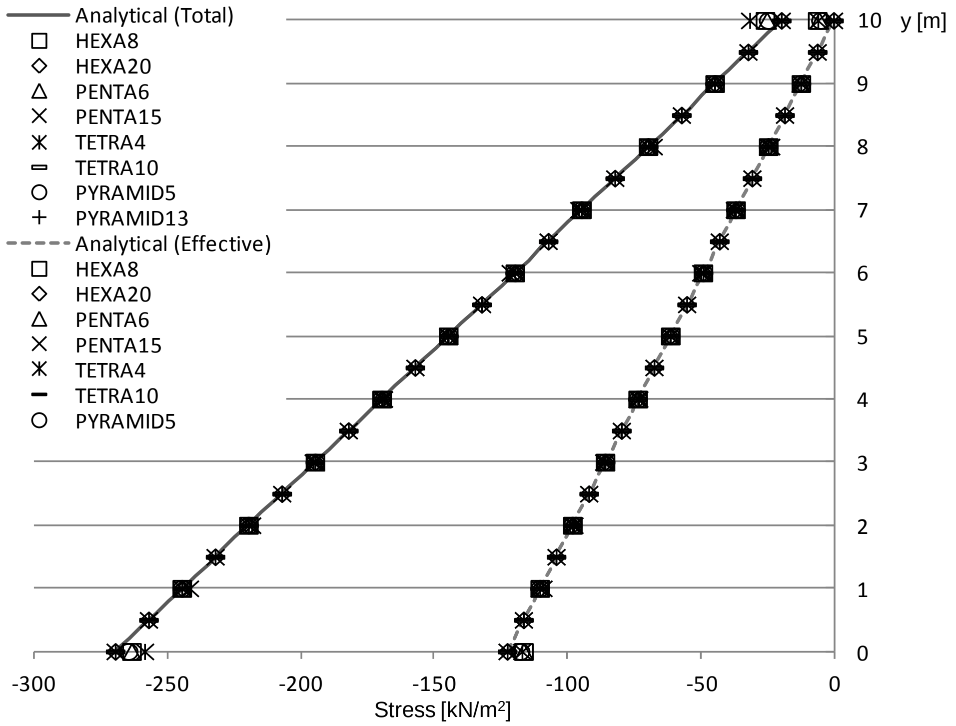


Figure 5.10.3
Total and effective vertical stress distributions in saturated soil obtained using solid elements



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