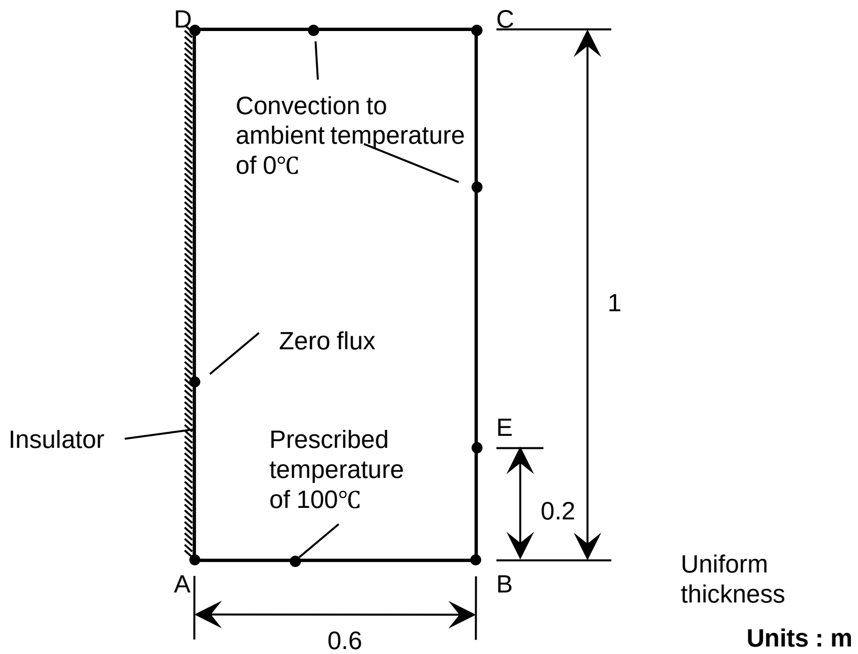
# 8.1 Two-dimensional heat transfer with convection

| | |
|---|---|
| **REFERENCE** | NAFEMS.[1] |
| **ELEMENTS** | Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer01_Steady.gts |

Figure 8.1.1 shows a two-dimensional heat transfer problem. The temperature of 100°C is prescribed to the edge AB. On the edges BC and CD, convection boundary conditions are applied with an ambient temperature at 0oC. The edge DA is insulated. Steady-state heat transfer analysis is carried out, and the temperature at the point E is determined.

Figure 8.1.1
Rectangular plate model




| | | |
|---|---|---|
| **Material data** | *Conductivity* | $k = 52.0\ J/m \cdot hr\ °C$ |
| | *Convection coefficient* | $h = 750.0\ W/m^2\ °C$ |

*Table 8.1.1 Temperature T at node E obtained using shell elements*

| | | $T_E$ [° C] |
|---|---|---|
| Reference | | 18.3 |
| Element type | Number of elements | |
| Tria-3 | 6x(10x2) | 18.9 |
| Quad-4 | 6x10 | 18.9 |
| Tria-6 | 3x(5x2) | 18.0 |
| Quad-8 | 3x5 | 18.0 |

*Table 8.1.2 Temperature T at node E obtained using solid elements*

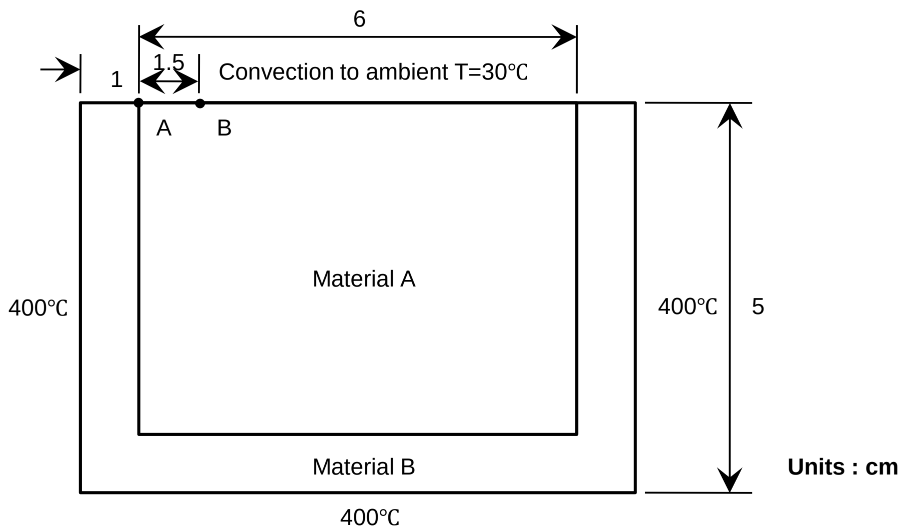
| | | $T_E$ [° C] |
|---|---|---|
| Reference | | 18.3 |
| Element type | Number of elements | |
| Penta-6 | 6x(10x2)x1 | 18.9 |
| Hexa-8 | 6x10x1 | 18.9 |
| Penta-15 | 3x(5x2)x1 | 18.0 |
| Hexa-20 | 3x5x1 | 17.9 |

# 8.2 Two-dimensional heat transfer in bi-material

| | |
|---|---|
| **REFERENCE** | Holman[2] |
| **ELEMENTS** | Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer02_Steady.gts |

Figure 8.2.1 shows a bi-material embedded in a high-thermal-conductivity material maintained at 400 ° C. The upper surface is exposed to a convection environment at 30° C. The temperature at the points A and B are determined and compared with the referenced solution given in "Holman"

Figure 8.2.1
Bi-material embedded in high-conductivity material




| | | |
|---|---|---|
| **Material data** | *Conductivity* | *k = 2.0 W/m °C (Material A)* |
| | | *k = 0.3 W/m °C (Material B)* |
| | *Convection coefficient* | *h = 25.0 W/m² °C* |

*Table 8.2.1 Temperature T at nodes A and B obtained using shell elements*

| | $T_A$ [°C] | $T_B$ [°C] |
|---|---|---|

| Reference | | 254.96 | 247.64 |
|---|---|---|---|
| Element type | Number of elements | | |
| Tria-3 | 12x(10x2) | *250.97* | *246.51* |
| Quad-4 | 12x10 | *250.07* | *246.49* |
| Tria-6 | 12x(10x2) | *249.86* | *246.35* |
| Quad-8 | 12x10 | *249.72* | *246.38* |

*Table 8.2.2 Temperature T at nodes A and B obtained using solid elements*

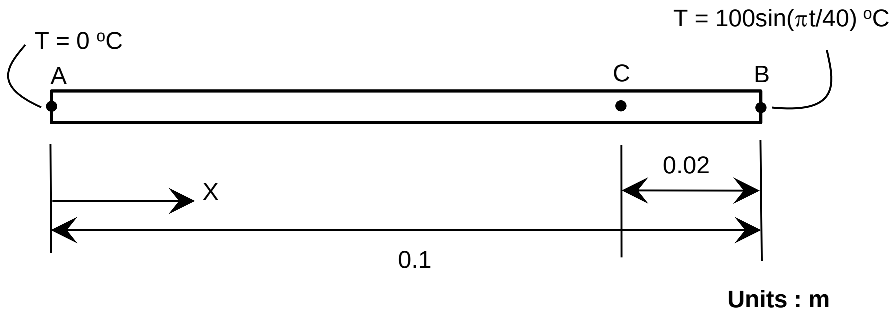
| | | $T_A$ $[°C]$ | $T_B$ $[°C]$ |
|---|---|---|---|
| Reference | | *254.96* | *247.64* |
| Element type | Number of elements | | |
| Penta-6 | 12x(10x2)x1 | *250.97* | *246.51* |
| Hexa-8 | 12x10x1 | *250.07* | *246.49* |
| Penta-15 | 12x(10x2)x1 | *249.72* | *246.38* |
| Hexa-20 | 12x10x1 | *249.86* | *246.35* |

# 8.3 One-dimensional transient heat transfer - I

| | |
|---|---|
| **REFERENCE** | NAFEMS[1] |
| **ELEMENTS** | Beam elements, Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer03_Transient.gts |

Figure 8.3.1 shows a one-dimensional transient heat transfer problem with conduction. A temperature of 0° C is assigned to the point A. A time variant temperature of $100\sin(\pi t/40)$ is set to the point B. The initial temperature of 0° C is applied at all of the nodes. Transient heat transfer analysis is carried out with a fixed time step of 1 second. The temperature at the point C at time t=32 sec is obtained using various finite elements. The solution from the NAFEMS benchmarks is taken as a reference for comparison.

Figure 8.3.1
One-dimensional transient heat transfer problem




| **Material data** | *Conductivity* | *k = 35.0 W/m °C* |
|---|---|---|
| | *Specific heat* | *C = 440.5 J/kg °C* |
| | *Density* | *$\rho$ = 7200 kg/m$^3$* |

*Table 8.3.1 Temperature at point C obtained using bar element*

| | | $T_C$ [°C] |
|---|---|---|
| Reference | | *36.60* |
| Element type | Number of elements | |
| Beam-2 | 10 | *35.51* |

*Table 8.3.2 Temperature at point C obtained using shell elements*

| | | $T_C$ [°C] |
|---|---|---|
| Reference | | *36.60* |
| Element type | Number of elements | |
| Tria-3 | 1x(10x2) | *35.51* |
| Quad-4 | 1x10 | *35.51* |
| Tria-6 | 1x(10x2) | *36.09** |
| Quad-8 | 1x10 | *36.09* |

* averaged temperature from nodes.

*Table 8.3.3 Temperature at point C obtained using solid elements*

| | | $T_C$ [°C] |
|---|---|---|
| Reference | | *36.60* |
| Element type | Number of elements | |
| Tetra-4 | 1x1x(10x2) | *35.51** |
| Pyram-5 | 1x1x10 | *35.86* |
| Penta-6 | 1x1x(10x2) | *35.51* |
| Hexa-8 | 1x1x10 | *35.51* |
| Tetra-10 | 1x1x(10x2) | *36.09** |
| Pyram-13 | 1x1x10 | *36.09** |
| Penta-15 | 1x1x(10x2) | *36.09** |
| Hexa-20 | 1x1x10 | *36.09* |

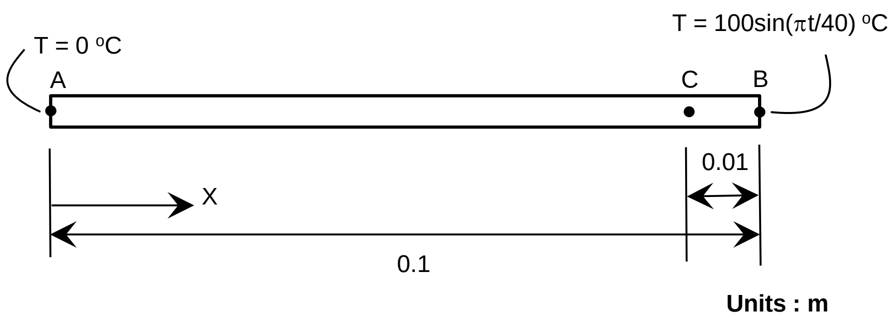
* averaged temperature from nodes.

# 8.4 One-dimensional transient heat transfer – II

| | |
|---|---|
| **REFERENCE** | NAFEMS[1], J. Barlow et al.[3] |
| **ELEMENTS** | Beam elements, Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer04_Transient.gts |

Figure 8.4.1 represents a one-dimensional transient heat transfer problem with conduction. The point A retains a fixed temperature of 0° C while the temperature at the point B varies with time, given by $100\sin(\pi t / 40)$. The entire model is at 0° C initially. Transient heat transfer analysis is carried out with a fixed time step of 1 second. The temperature at the point C at time t=58 sec is obtained using various finite elements. Due to the high level of temperature gradient near the point C, high order elements perform significantly better than low order elements.

Figure 8.4.1
One-dimensional transient heat transfer problem




| | | |
|---|---|---|
| **Material data** | *Conductivity* | *k = 1.0 W/m °C* |
| | *Specific heat* | *C = 985 J/kg °C* |
| | *Density* | *ρ = 2300 kg/m³* |

*Table 8.4.1 Temperature at point C obtained using bar elements*

| | | $T_C$ [°C] |
|---|---|---|
| Reference | | *9.62* |
| Element type | Number of elements | |
| Beam-2 | 10 | *3.46* |
| | 20 | *7.11* |

*Table 8.4.2 Temperature at point C obtained using shell elements*

| | | $T_C$ [°C] |
|---|---|---|
| Reference | | *9.62* |
| Element type | Number of elements | |
| Tria-3 | 1x(10x2) | *3.46* |
| | 1x(20x2) | *7.11* |
| Quad-4 | 1x10 | *3.46* |
| | 1x20 | *7.11* |
| Tria-6 | 1x(10x2) | *9.07** |
| | 1x(20x2) | *9.03** |
| Quad-8 | 1x10 | *9.36* |
| | 1x20 | *9.14* |

* averaged temperature from nodes.

*Table 8.4.3 Temperature at point C obtained using solid elements*

| | | $T_C$ [$^\circ C$] |
|---|---|---|
| Reference | | *9.62* |
| Element type | Number of elements | |
| Tetra-4 | 1x1x(10x2) | *3.46** |
| | 1x1x(20x2) | *7.05** |
| Pyram-5 | 1x1x10 | *6.94* |
| | 1x1x20 | *7.71* |
| Penta-6 | 1x1x(10x2) | *3.46* |
| | 1x1x(20x2) | *7.11* |
| Hexa-8 | 1x1x10 | *3.46* |
| | 1x1x20 | *7.11* |
| Tetra-10 | 1x1x(10x2) | *9.26** |
| | 1x1x(20x2) | *8.99** |
| Pyram-13 | 1x1x10 | *7.76** |
| | 1x1x20 | *9.15** |
| Penta-15 | 1x1x(10x2) | *9.00** |
| | 1x1x(20x2) | *8.99** |
| Hexa-20 | 1x1x10 | *9.36* |
| | 1x1x20 | *9.14* |

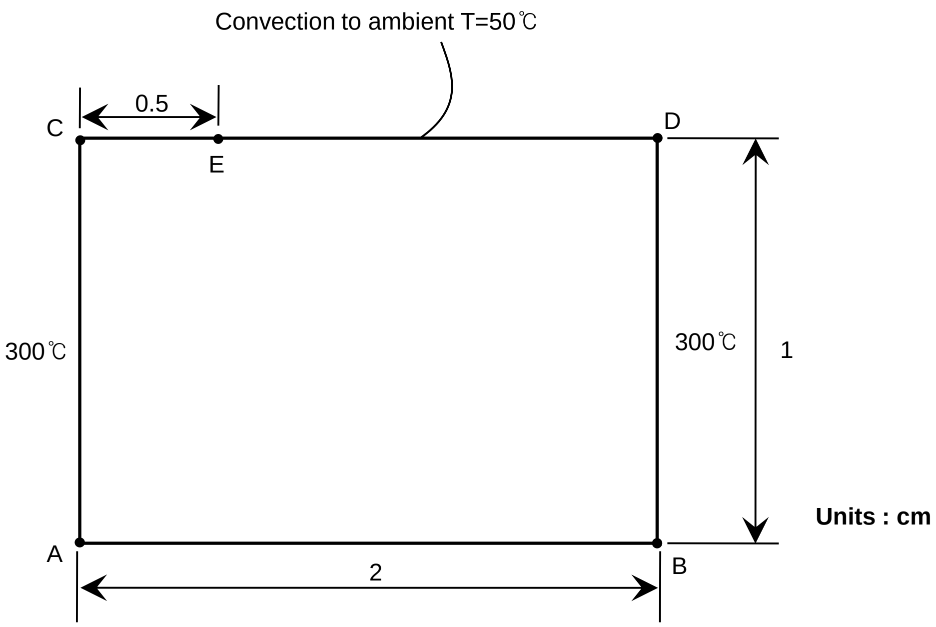
* averaged temperature from nodes.

# 8.5 Transient heat transfer with convection

| | |
|---|---|
| **REFERENCE** | Holman[2] |
| **ELEMENTS** | Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer05_Transient.gts |

Figure 8.5.1 shows a two-dimensional transient heat transfer model whose sides are maintained at a temperature of 300° C. The bottom edge of the model is insulated, and the top edge is subjected to heat exchange by convection. The ambient temperature for convection is 50° C, and the convective heat transfer coefficient is 200W/m$^2$·°C. The entire model is at 300° C initially. The temperature at the point E after 12 seconds is obtained through transient heat transfer analysis with a time step of 2 seconds.

Figure 8.5.1
Two-dimensional transient heat transfer problem




| | | |
|---|---|---|
| **Material data** | *Conductivity* | *k = 3.0 W/m °C* |
| | *Specific heat* | *C = 800 J/kg °C* |
| | *Density* | *ρ = 1600 kg/m³* |

*Table 8.5.1 Temperature at point E obtained using shell elements*

| | | $T_E$ $[°C]$ |
|---|---|---|
| Reference | | *243.32* |
| Element type | Number of elements | |
| TRIA-3 | 20x(10x2) | *239.71* |
| QUAD-4 | 20x10 | *239.56* |
| TRIA-6 | 20x(10x2) | *239.46* |
| QUAD-8 | 20x10 | *239.46* |

*Table 8.5.2 Temperature at point E obtained using solid elements*

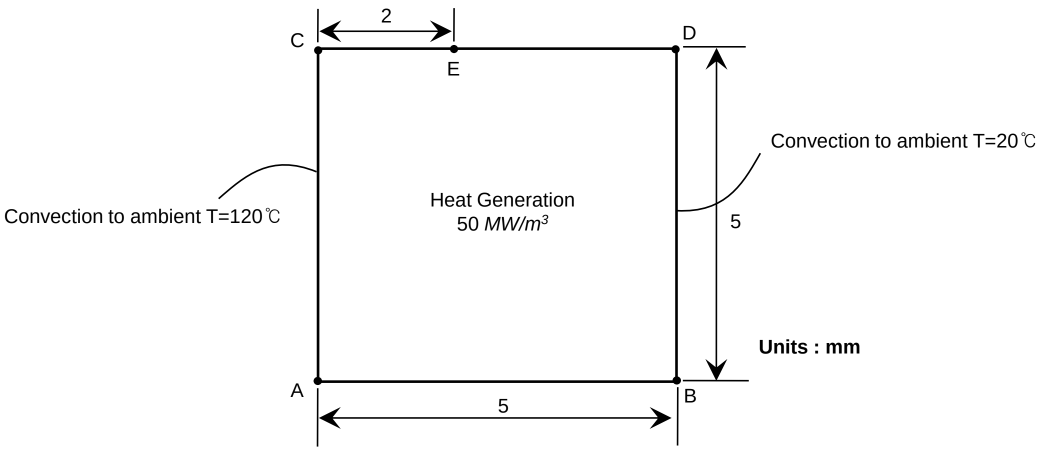
| | | $T_E$ $[°C]$ |
|---|---|---|
| Reference | | *243.32* |
| Element type | Number of elements | |
| TETRA-4 | 20x(10x24)x1 | *239.56* |
| PYRA-5 | 20x(10x6)x1 | *239.56* |
| PENTA-6 | 20x(10x2)x1 | *239.71* |
| HEXA-8 | 20x10x1 | *239.56* |
| TETRA-10 | 20x(10x24)x1 | *240.63* |
| PYRA-13 | 20x(10x6)x1 | *239.46* |
| PENTA-15 | 20x(10x2)x1 | *239.46* |
| HEXA-20 | 20x10x1 | *239.46* |

# 8.6 Transient heat transfer with heat generation

| | |
|---|---|
| **REFERENCE** | Holman[2] |
| **ELEMENTS** | Shell elements, Solid elements |
| **MODEL FILENAME** | HeatTransfer06_Transient.gts |

Figure 8.6.1 represents a two-dimensional plane with 50MW/m$^3$ internal heat generation. The two side edges are subjected to convection boundary conditions. The edges AC and BD retain the convection coefficients of 400W/m$^2$ and 500 W/m$^2$ respectively. The ambient temperatures for the two edges are 120° C and 20° C respectively. The plane is initially at a uniform temperature of 100° C. The temperature at the point E after 9 seconds is obtained by transient heat transfer analysis.

Figure 8.6.1
Two-dimensional transient heat transfer problem




| | | |
|---|---|---|
| **Material data** | *Conductivity* | *k = 19.0 W/m °C* |
| | *Specific heat* | *C = 460 J/kg °C* |
| | *Density* | *ρ = 7800 kg/m³* |

*Table 8.6.1 Temperature at point E obtained using shell elements*

| | | $T_E$ [$^\circ C$] |
|---|---|---|
| Reference | | *190.70* |
| Element type | Number of elements | |
| TRIA-3 | 5x(5x2) | *190.37* |
| QUAD-4 | 5x5 | *190.39* |
| TRIA-6 | 5x(5x2) | *190.32* |
| QUAD-8 | 5x5 | *190.32* |

*Table 8.6.2 Temperature at point E obtained using solid elements*

| | | $T_E$ [$^\circ C$] |
|---|---|---|
| Reference | | *190.70* |
| Element type | Number of elements | |
| PENTA-6 | 5x(5x2)x1 | *190.37* |
| HEXA-8 | 5x5x1 | *190.39* |
| TETRA-4 | 5x(5x24)x1 | *190.37** |
| PYRA-5 | 5x(5x6)x1 | *190.38* |
| PENTA-15 | 5x(5x2)x1 | *190.32* |
| HEXA-20 | 5x5x1 | *190.32* |
| TETRA-10 | 5x(5x24)x1 | *190.32* |
| PYRA-13 | 5x(5x6)x1 | *190.32* |

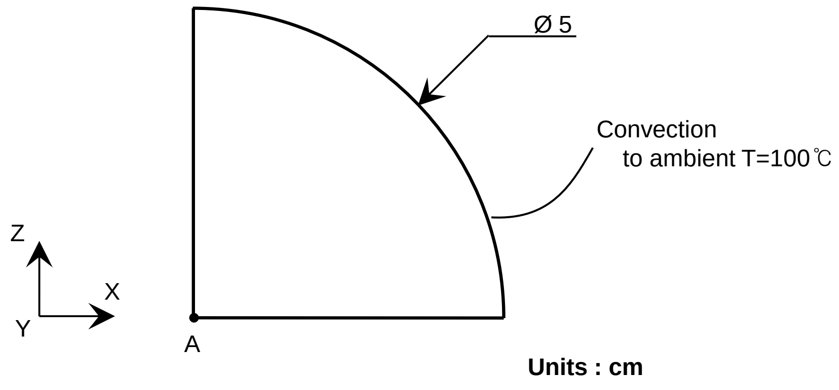
* averaged temperature from nodes.

# 8.7 Axisymmetric transient heat transfer with convection

| | |
|---|---|
| **REFERENCE** | Holman[2] |
| **ELEMENTS** | Axisymmetric elements |
| **MODEL FILENAME** | HeatTransfer07_Transient.gts |

Figure 8.7.1 shows a 5.0 cm diameter hemisphere with an initial temperature of 450°C. The hemisphere is subjected to convection with an ambient temperature of 100°C and a convection coefficient of 10 W/m$^2$·°C. Axisymmetric heat transfer elements are employed to evaluate the transient response of the model. The temperature obtained at the point A at time t=5819 sec is compared with the reference value.

Figure 8.7.1
Axisymmetric transient heat transfer problem




| | | |
|---|---|---|
| **Material data** | *Conductivity* | *k = 35.0 W/m °C* |
| | *Specific heat* | *C = 460 J/kg °C* |
| | *Density* | *$\rho$ = 7800 kg/m$^3$* |

*Table 8.7.1 Temperature at point A obtained using axisymmetric elements*

| | | $T_A$ [$^\circ C$] |
|---|---|---|
| Reference | | *150.0* |
| Element type | Number of elements | |
| TRIAX-3 | 196 | *151.06* |
| QUADX-4 | 94 | *151.06* |
| TRIAX-6 | 196 | *151.06* |
| QUADX-8 | 94 | *151.06* |

# References

1 NAFEMS, The Standard NAFEMS Benchmarks, Rev. 3, NAFEMS, Glasgow, 1990

2 J. P. Holman, Heat Transfer, 9th Edition, McGraw-Hill, 2002

3 J. Barlow, G. A. O. Davis, Selected FE Benchmarks in Structural and Thermal Analysis, NAFEMS, Glasgow, 1987