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AdSec

Oasys AdSec

Theory

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Oasys AdSec

Contents

Introduction

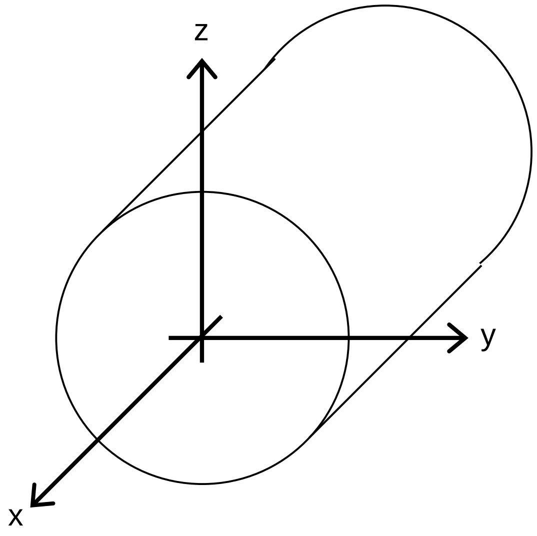
AdSec is a general section analysis program, giving section properties for irregular sections. While focussed on concrete sections, other materials can be specified. It will:

- Calculate the ultimate resistance of irregular sections (reinforced & pre-stressed)
- Provide N/M & M_{yy}/M_{zz} charts
- Serviceability calculations including crack widths / cracking moment
- Provide moment–curvature and moment–stiffness charts

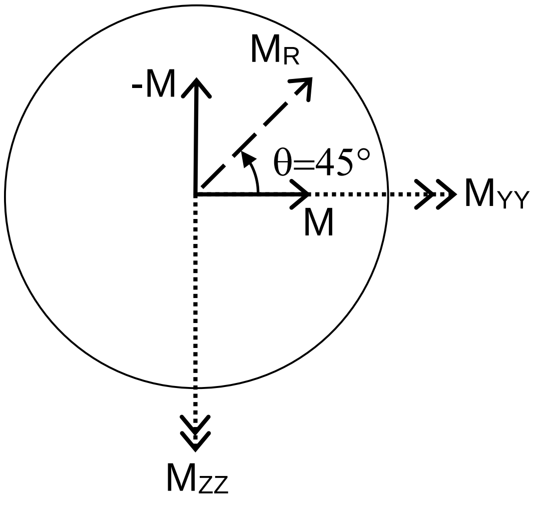
The basic assumption in AdSec is that plane sections remain plane.

Sign Convention

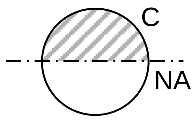
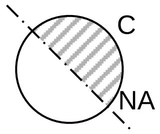
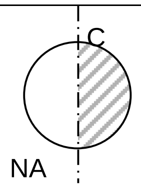
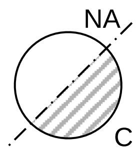
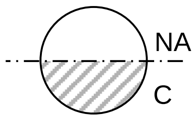
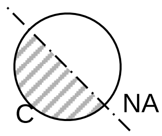
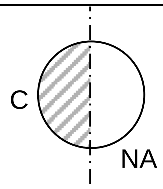
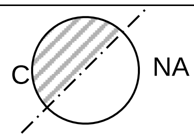
In AdSec the section is considered to be in the y-z plane as illustrated below



This means that moments are defined as follows:



The following table illustrate moment, compression and curvature conventions

M_{yy}	M_{zz}	Moment angle, θ	Neutral Axis angle, θ_{NA}	κ_{yy}	κ_{zz}	
+ M	0	0	0	+ ve	0	
+ M	+ M	-45°	-45°	+ ve	+ ve	
0	+ M	-90°	-90°	0	+ve	
- M	+M	-135°	-135°	- ve	+ ve	
- M	0	-180° (180°)	-180° (180°)	- ve	0	
- M	- M	135°	135°	- ve	- ve	
0	- M	90°	90°	0	- ve	
+ M	- M	45°	45°	+ ve	- ve	

Key: C Region of Compression

NA Neutral Axis

M Any particular moment

Analysis options

For the Ultimate Limit State, the options available include:

- Ultimate moment capacity of the section.
- Stresses and strains from the ultimate applied load and stresses at the ultimate limit state.
- Ultimate resistance N/M interaction chart.
- Ultimate resistance M_{yy}/M_{zz} moment interaction chart.

For the Serviceability Limit State, the program calculates:

- Cracking moment.
- Stresses, stiffness and crack widths for each analysis case.
- Moment-stiffness and moment-curvature charts.

The following load types can be simulated:

- Pre-stress using unbonded tendons
- Shrinkage and temperature effects

AdSec will find the ultimate capacity of a simple or compound section. It will find the state of stress & strain in the section under a variety of loading conditions for serviceability and ultimate material properties. Serviceability analysis will generate a plot of neutral axis position and crack widths around the section as well as full numerical output

The ultimate capacity charts have developed significantly. The user can specify a table of additional points (N/M or Myy/Mzz), with labels, which will be plotted onto the graph. Also the user can specify a number of values of axial force and Myy/Mzz plots will be drawn for each value of axial force on the same graph. User input strain planes can applied to see the impact on the ultimate capacity charts.

Serviceability charts will plot moment versus curvature, secant stiffness or tangent stiffness for a given value or range of axial force and moment angle.

Loading

The reference point for loading and strain planes is taken by default as the Geometric Centroid, but this can be overridden by specifying a user specified point.

Solution

The basic idea behind AdSec is that the state of strain across a section varies linearly and can be defined by a 'strain plane'. As the variation is linear the strain plane can be define by a scalar axial strain (applied at a specified point) and a curvature vector.

$$\bar{\varepsilon} = \lfloor \varepsilon_x, \kappa_y, \kappa_z \rfloor$$

Then the axial force and moments in the section are then defined as

$$N_x = \int_A \sigma(\varepsilon) dA$$
$$M_y = \int_A \sigma(\varepsilon) z \, dA$$
$$M_z = \int_A \sigma(\varepsilon) y \, dA$$

This can be characterized in the same way as the strain plane as

$$\overline{N} = [N_x, M_y, M_z]$$

There are many different calculations in AdSec but they are all defined as solution to a set of equations which are functions of the strain plane.

When checking the strength of a section given the axial force and a moment vector, the solution is to find the maximum moment possible before the section fails. There are several criteria to be checked but the primary criteria is

$$N_x = N_{x,app}$$
$$\theta_M = \theta_{M,app}$$
$$\varepsilon_{\max} = \varepsilon_u$$

Where N_x is the axial force, θ_M is the moment angle, $\varepsilon_{\max}$ is the maximum strain in the section, ε_u is the ultimate (failure) strain of the material and the subscript 'app' is for applied.

The basic solution procedure is select an initial trial strain and calculate the target values.

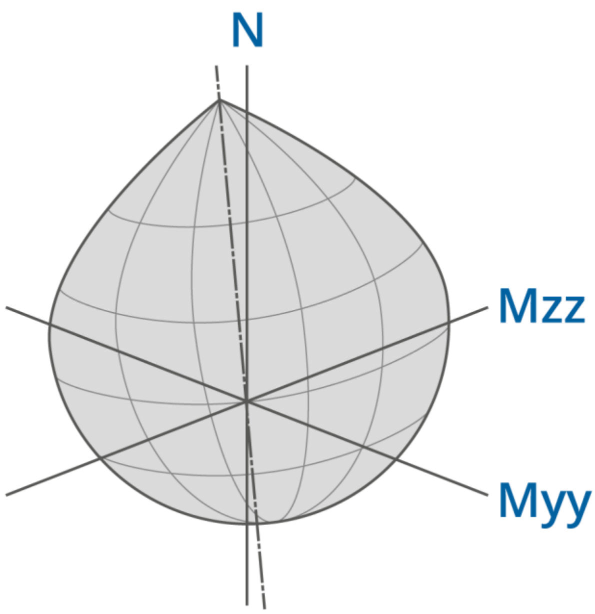
$$N_x = f_1(\bar{\varepsilon})$$
$$\theta_M = f_2(\bar{\varepsilon})$$
$$\varepsilon_{\max} = f_3(\bar{\varepsilon})$$

The solution is then iterative until a converged solution for the criteria above is achieved. Once this strain plane is found the ultimate moment M_u is the moment from integrating the final strain plane.

Solver Solution Space

Any particular load condition can be considered as a point in the $\left(N, M_{yy}, M_{zz}\right)$ space. It is them possible to construct a failure envelope – this can be for ULS strength or for a serviceability criteria, such as cracking. The failure envelope is then typically an 'onion' shape with axial force as the 'vertical' axis. Any point inside the space represent a valid force state, but a point outside the surface has failed.

The analysis is then concerned with determining the force position with respect to the failure surface. In a ULS analysis the solution holds the axial force constant and find the ratio of the applied moment to the moment corresponding to the projection on to the failure surface.



Similarly the chart options N/M corresponds to a vertical slice through the 'onion' and a horizontal slice corresponds to an M_{yy}/M_{zz} chart.

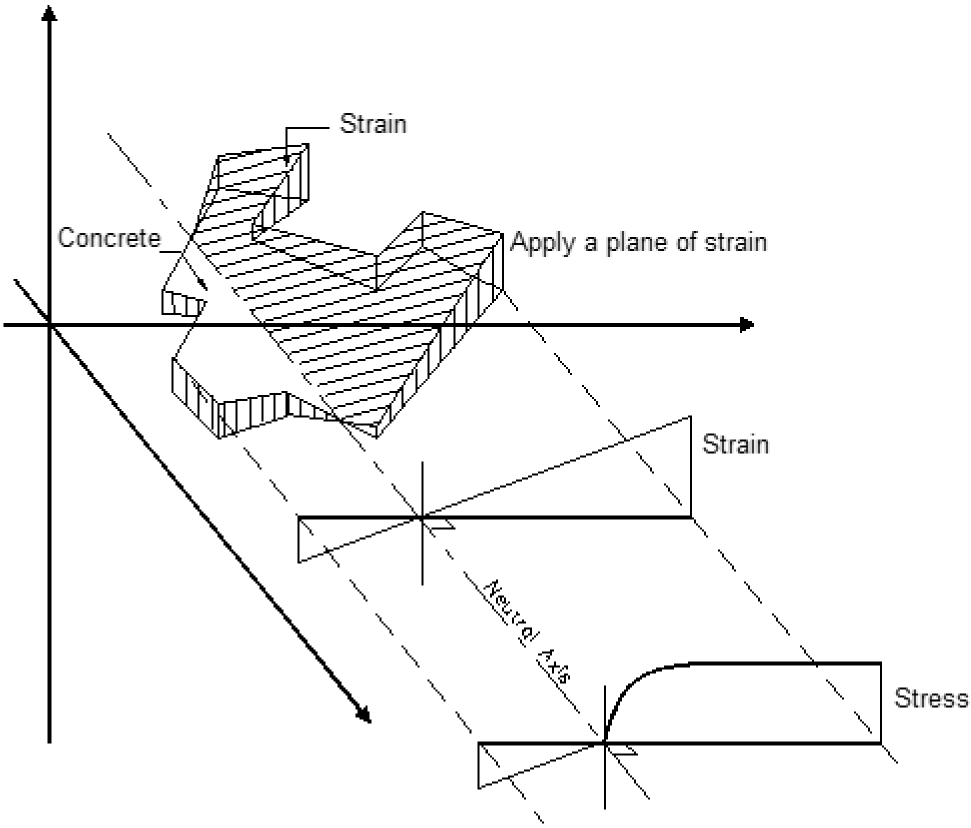
Solver Search Process

The solution is iterative, and assumes plane sections remain plane. The iteration searches through possible strain planes. Strain ε at point $\left(y, z\right)$ is $\varepsilon_x + \kappa_z y + \kappa_y z$. For each strain plane: locked in strains are added, stresses are calculated from the non-linear material curves, forces and moments are calculated by integration of stresses over the section The three search conditions are then checked.

Analysis	Condition 1	Condition 2	Condition 3
ULS : Strength	Axial force	Moment angle	Ultimate strain

ULS : N/M charts	Moment angle	Strain condition	Ultimate strain
SLS & ULS : Loads	Axial force	Moment angle	Applied moment
SLS : Cracking moment	Axial force	Moment angle	Cracking strain

The iteration continues until the three variables $\varepsilon_x, \kappa_z, \kappa_y$ are found with form a strain plane $\hat{\varepsilon}$ that satisfies the (up to) three conditions.



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SLS and ULS loads analysis

For a loads analysis AdSec searches for a strain plane which satisfies:

- axial force = applied axial force
- moment = applied moment
- moment angle = applied moment angle

When looking at serviceability it is useful to be able to differentiate between short and long term conditions. Long term analysis takes account of creep, while short term analysis assumes that no creep takes place.

ULS strength analysis

For a strength analysis there are several criteria that may govern the strength of the section. For a ‘concrete’ (compression) governed section

- axial force = applied axial force
- moment angle = applied moment angle
- concrete strain : concrete failure strain = 1

For a ‘reinforcement’ (tension) governed section

- axial force = applied axial force
- moment angle = applied moment angle
- rebar strain : rebar failure strain = 1

Some codes implement a third condition which limits the compressive strain on the section when the whole section is in compression.

Long, short and long & short term analysis

When looking at serviceability it is useful to be able to differentiate between short and long term conditions. Long term analysis takes account of creep, while short term analysis assumes that no creep takes place.

The creep is defined by a creep coefficient φ for concrete. It is assumed that the other materials are unaffected by creep. This coefficient is used to modify the material stress-strain curves.

In a long term analysis the total strain is assumed to include the strain due to load plus an additional strain due to creep. In the linear case this can be written as

$$\varepsilon = \varepsilon_{\text{load}} + \varepsilon_{\text{creep}} = \frac{\sigma}{E} + \varphi\frac{\sigma}{E}$$

Rearranging this gives

$$\sigma = \varepsilon\frac{E}{(1+\varphi)}$$

AdSec takes these creep effects into account by modifying the effective elastic modulus

$$E_{\text{creep}} = \frac{E}{(1+\varphi)}$$

Resulting in stress-strain curves stretched along the strain axis.

'Long and short' term analysis is an option in AdSec to give a more detailed understand the serviceability behaviour of sections. Note: this is not available for all design codes.

Loading is defined in two stages. Firstly long term loading, combined with a creep factor and then an additional short term loading.

In some circumstances, the long term loading is a permanent or quasi permanent loading, and the short term loading is an extreme event that happens after an extended period of time. However in many cases short term loading will occur intermittently throughout the life of the section. The long and short term analysis option in AdSec will model the second case.

- Firstly the cracking moment is calculated assuming the total long & short term axial load and moment direction, and short-term material properties.
- Secondly the strains and stresses are found for the long term loads, and long term material properties. These strains are used to calculate the creep effects of long term loads where creep strain is: $\varepsilon_{\text{creep}} = -\varepsilon_{\text{long}} \dfrac{\varphi}{(1+\varphi)}$. In this analysis the BS8110 Pt.2 tension curve will use 0.55N/mm^2 as the maximum stress, if the section was deemed cracked under the total load. This will model the conservative assumption that, if cracked, this happened at an early stage of the sections life.
- The cracking moment is then recalculated, for the total load including the creep strain in the concrete calculated above. This will have the effect of slightly reducing the cracking moment if a compressive force has been acting on the section for a long time. This is the case, because the stress in the concrete will have reduced as the concrete creeps and more stress is transferred to the reinforcement.
- Finally a short term analysis is performed for the total loads, using short term material properties and the calculated creep strain to include for the long term effects.

Note that if the same process is followed manually using sequential AdSec analyses the initial cracking moment will be calculated from the long term load only. This will give different results than the automated AdSec 'long and short' term analysis in a small number of cases. The cases affected are where the BS8110 Pt.2 tension curve is selected, and the section is cracked under total load, but uncracked under the long term load, and the stress under long term load is between 0.55 and 1.0 N/mm^2 at the centroid of tension steel.

Some codes allow an intermediate term analysis, depending on the ratio

$$\frac{M_q}{M_g}$$

In this case

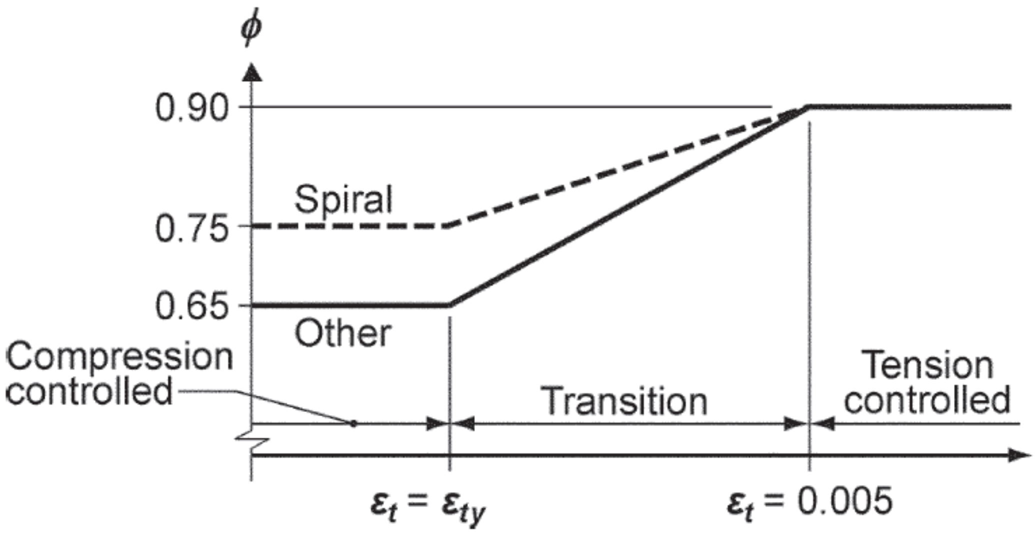
$$E_{\text{inter}} = E_{short} \frac{(M_q/M_g)}{1+(M_q/M_g)} + E_{long} \frac{1}{1+(M_q/M_g)}$$

$$E_{short} = E$$

$$E_{long} = \frac{E}{(1+\varphi)} = \frac{E}{2}$$

Strength reduction or material partial factors for ULS

There are two main approaches applicable to section analysis: strength reduction and material partial factors. The strength reduction approach is use in the ACI (American) and AS (Australian) codes. In this case the material strengths are used unfactored, the strength of the section is calculated and then reduced using a strength reduction factor φ. The particular value of φ depends on the strain place at failure.



In most other codes factors are applied to the material in the ULS. Giving a reduced strength for the material in design

$$f_d = \frac{f}{\gamma_m}$$

The CSA (Canadian) codes use a similar approach but use a factor φ_m.

$$f_d = \varphi_m f$$

The values of γ_m and φ_m are specified in the code but typical values are

	γ_m	φ_m
Concrete	1.5	0.65
Reinforcement	1.15	0.85
Pre-stress tendons	1.15	0.9
Structural steel		0.9

Tension in Concrete

Concrete exhibits a 4 phase behaviour in response to tension stresses.

- Low tension stress – concrete tension stiffness similar to compression
- Cracking starts – stiffness drops off as cracks form
- Cracks formed, cracks open up – stiffness drops off more rapidly as cracks open up
- Fully cracked – no residual stiffness left

This behaviour is complex as it is controlled by the reinforcement. The simplified means prescribed to deal with these phenomena vary from code to code.

All codes state that ultimate analysis and design should ignore the tension stiffening from the concrete. All codes will accept fully cracked section properties as a lower bound on stiffness.

Serviceability analysis is usually performed for stiffness, stress/strain checks, or crack width checks. Some codes imply a different tension stiffening method for crack width as opposed to the other checks. This may lead to a disparity in AdSec results between the 'cracking moment' and the moment at which the crack width becomes > zero.

The code rules are developed for a rectangular section with uniaxial bending and one row of tension steel. However the rules are not extended to sections made up of various zones of concrete, some with locked in strain planes. Because the tension stiffening is a function of the amount of 'damage' / cracking in the section, adjoining tensile zones need to be considered in evaluating the tension strength of a zone, as these may contain steel which will control the cracking.

BS8110 Pt 2 presents a stress/strain 'envelope' which provide means of calculating an effective tensile Young's modulus for a linear tension stress/strain curve.

ICE Technical note 372 presents a more sophisticated envelope approach than BS8110 and is offered as an option in ADSEC.

BS5400 presents the same approach as BS8110 in Appendix A for stiffness calcs. But this is rarely used. Instead the main body of the code gives a crack width formula based on strains from an analysis with no tension stiffening. The crack width formula itself includes some terms to add back in an estimate of the contribution from tension stiffening. Ref BS5400 5.8.8.2 equation 25.

EC2 proposes 2 analyses, one with full tension stiffness and one with none. The final results are an interpolation between these results.

Recent research about the cracked stiffness of concrete has shown that the tension stiffness measured in the laboratory can only be retained for a very short time. This means that both the tension stiffening given in BS8110 and TN 372 is un-conservative for most building and bridge loadings. AdSec includes these findings for BS8110 and will give a smaller tension stiffness than previous versions.

EC2 tension stiffening

EC2 tension stiffening is described in Eurocode 2 section 7.4.3 equation 7.18. EC2 does not have a specific tension stiffening relationship used in analysis. Instead, two analyses are carried out assuming cracked and uncracked stiffness values, and the actual curvature & stiffness is an interpolation between the 2 results based on the amount of cracking predicted.

The cracking moment, M_{cr}, is defined as the moment when the stress in the outer most tensile element of an uncracked concrete section has reached f_{ctm}.

The tension stiffening options offered for EC2 in AdSec are zero tension, linear tension, and interpolated. 'Zero-tension stiffness' will give a conservative, fully cracked lower bound. The 'linear tension stiffening' uses the Elastic modulus of the concrete to produce a linear stress–strain relationship. This is for checking of the other results only and it is not appropriate to use this beyond the cracking moment. Note that the values in EC2 for serviceability are based on mean concrete properties rather than the characteristic values used for ultimate analysis and design. The interpolation depends on the amount of damage sustained by the section. This is calculated by AdSec based on the proximity of the applied loading to the cracking moment. But for sections which have been cracked in a previous load event the minimum value of ζ for use in equation 7.18 can be input. The default value of $\zeta_{\min}$ is 0. To take account of the fast drop in tension stiffening following cracking, the value of β in equation 7.19 defaults to 0.5.

AdSec does not use equation 7.19 to calculate the damage parameter ζ. Instead ζ is calculated from the cracking strain

$$\varepsilon_{cc} = \frac{f_{ctm}}{Ecm}$$

and the most tensile strain ε_{uncr} in the section under an uncracked analysis under full applied load.

The E used for to determine ε_{uncr} is short term (not modified for creep). For composite sections ζ_i is calculated for each component i using the component material properties for ε_{cc}, and the most tensile strain on the component for ε_{uncr}. The highest value of ζ_i will be used for ζ in stiffness & cracking calculations.

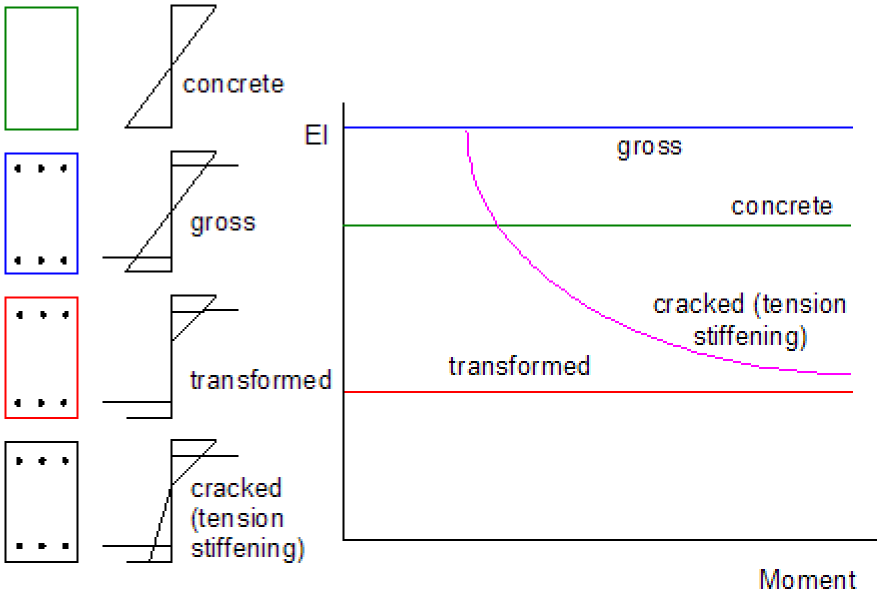
Note engineering judgement should be used to assess if this approach fits the particular situation.

$$\zeta = \begin{cases} 1-\beta\left(\dfrac{\varepsilon_{cc}}{\varepsilon_{uncr}}\right)^2 & \varepsilon_{uncr} > \varepsilon_{cc} \\ \zeta_{\min} & \varepsilon_{uncr} \le \varepsilon_{cc} \end{cases}$$

If the EC2 interpolation is selected for the tension stiffness at serviceability, the properties which depend on the average behaviour along the element (eg stiffness, curvature and crack widths) are based on the interpolated strain plane. However for moments greater than M_{cr}, the stresses output by AdSec for the interpolated tension stiffness are from the fully cracked analysis, because these represent the maximum stresses which occur at crack positions.

Stiffness

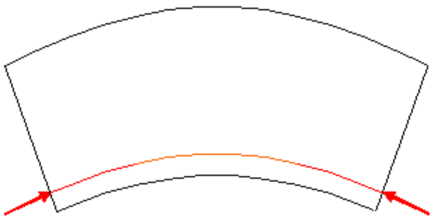
AdSec operates on strain, using non-linear materials. AdSec will show how the stiffness of the section changes with load and the effect of non-linear material behaviour. There are a number of ways in which the stiffness of a reinforced concrete section can be approximated. These are show in the diagram below. This diagram plots AdSec results along with the approximate stiffness values for comparison



For a symmetric section, symmetrically loaded, stiffness can be expressed as

$$EI = \frac{M}{\kappa}$$

If there is an axial force, locked in strain plane, or pre-stress, there will be a residual curvature at zero moment.



This curvature can be called κ_0 so AdSec uses

$$EI = \frac{M}{(\kappa - \kappa_0)}$$

The curvature at zero moment may not be in the same direction as the applied moment angle. To allow for this, the formula is further modified to give

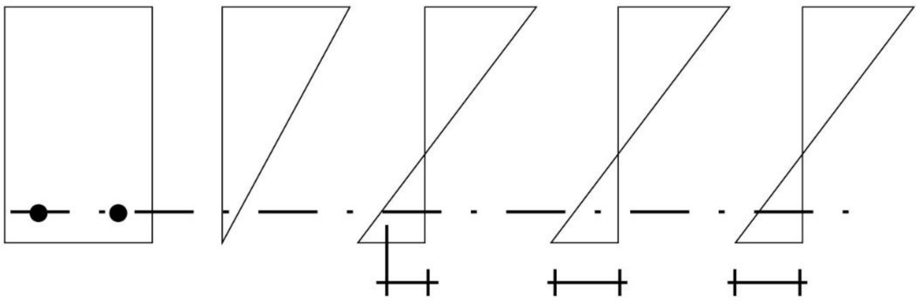
$$EI = \frac{M}{[\kappa - \kappa_0(\alpha_{\text{appl}} - \alpha_{\text{NA}})]}$$

Where α_{appl} is the angle of applied moment and α_{NA} is the neutral axis angle from the κ_0 calculation.

Cracking

Cracking Moment

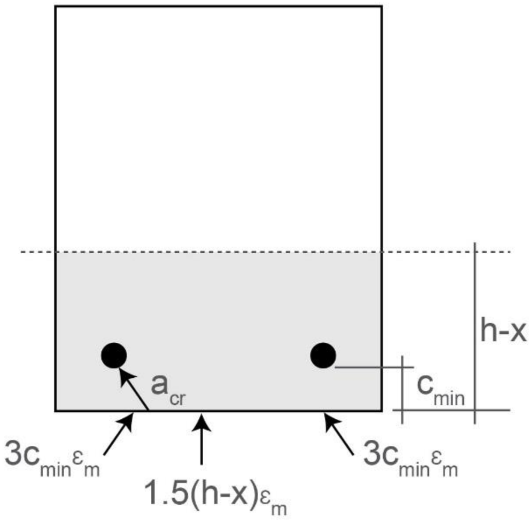
The programme calculates the total axial force and moment from the loads in the analysis case definition. It uses the axial force and the angle of the applied moment to define the cracking moment analysis task. The programme then searches for a strain plane that gives the cracking strain as shown below.



Integrating the stresses from this strain plane over the section will give an axial force equal to the applied force, and a moment which is parallel to the applied moment. The value of this moment is the cracking moment. Short-term material properties are always used for the cracking moment calculation.

Crack-width

Crack width formulae to BS8110 and BS5400 are based on a weighted interpolation between two effects. Close to a bar, crack width is a function of the bar cover, $c_{\min}$. Between bars, it is a function of the depth of tension zone, $h-x$.



BS8110 & Hong King Code of Practice

Crack widths can be calculated to BS8110 using any of the three available tension stiffening options.

After calculating the cracking moment, AdSec will search for a strain plane which gives forces and moments within tolerance of the applied forces and moments. The resulting strain distribution is used to calculate the crack width.

The maximum crack width output is related to the given resultant moment orientation. This is particularly important for circular sections, as the maximum strain may not occur between two bars.

This would give a lower crack width value than may occur in reality.

The sides of the section are divided into small segments and the crack width calculated for each segment. The crack width formula

$$w = \frac{3a_{cr}\varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{\min}}{h - x}\right)}$$

is given in BS8110, section 3.8, equation 12.

Crack width calculations involve a large amount of engineering interpretation for faceted sections, sections with voids, sections with re-entrant corners, and multi-zone sections. Depending on the situation, a different definition of 'cover' is required. The programme stores the minimum cover to each bar and uses this in the calculation of crack width. This means that the cover used in the calculation may not relate to the side being checked (it will always give a conservative result). The reason for this approach is that curved sections are analysed as a multifaceted polygon, and there may be no bars present parallel to a small facet. This is because the number of facets may be greater than the number of bars.

The crack width calculation is done on a zone by zone basis using the zonal strain plane (resulting strain plane + component strain plane + concrete only component strain plane). This component strain plane applied to the whole section is used to calculate neutral axis depth and section height (x and h) relative to the whole section – using all the section coordinates.

The crack width includes the term $(a_{cr} - c_{\min})$. If $c_{\min}$ is smaller than a_{cr} the crack width is increased.

For each division on the concrete outline the closest bar is found (minimum a_{cr}). For a re-entrant corner, and a bar which is on the 'outside' of the section with ref to the side being checked a warning flag is generated, A conservative crack width can still be calculated using the minimum cover to the bar.

If the cover is greater than half the depth of the tension zone, the crack width in both codes in invalid.

The term for concrete only cracking should be used instead. This is $1.5(h-x)\varepsilon_m$. This is included in AdSec.

These are warning in the crack width calculation. They do not necessarily mean that the answer is wrong. But do mean that the graphical results should be checked for engineering interpretation

- c_{min} < controlling bar diameter – crack width not valid
- Controlling bar is remote from crack location
- Controlling bar and crack are located on either side of re-entrant corner
- Cover to controlling bar measured to different side from crack location

BS540, Hong Kong Strucutres Dsign Manual & IRS Bridge Code

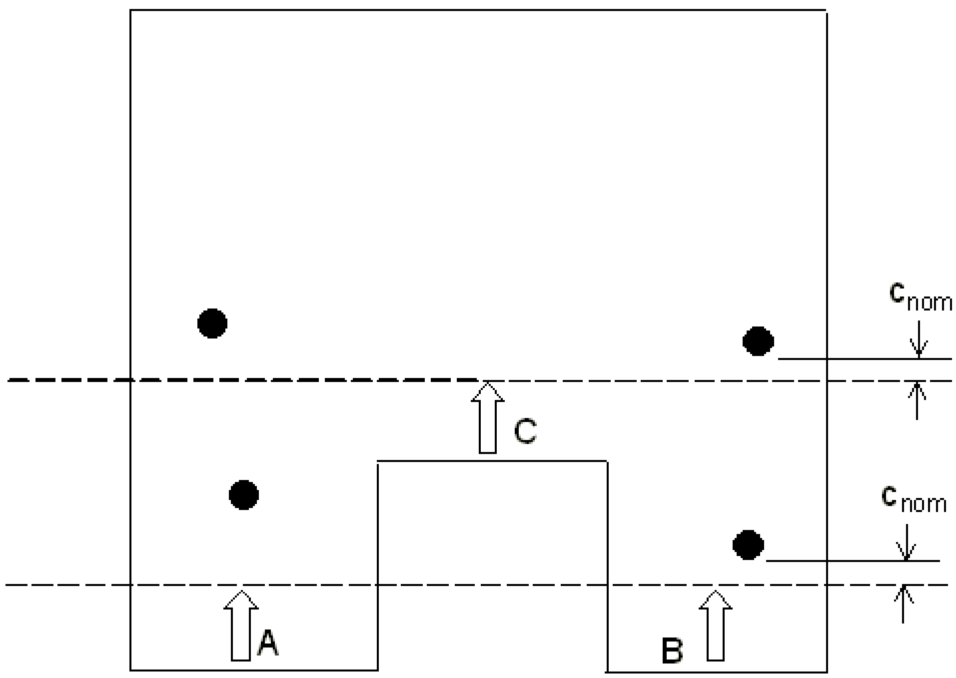
The BS8110 specification above is valid for BS5400 plus some additional points.

The BS5400 includes a fudge for effective tension stiffening. So for crack widths to be correct using the BS5400 formula, SLS analysis must use no tension stiffening in calculation of the strains around the section.

This fudge requires calculation of 'the level of tension steel'. This is re-calculated on a component by component basis using a similar method to BS8110-2 tension stiffening. The tension steel is identified as the steel which is in the tension zone when the zonal strain plane is extended across the whole section (the zonal strain plane is the resulting strain plane + zonal locked in plane + zonal concrete only plane). From the steel bars identified, the centroid of steel force is calculated using the actual stress in the bars ignoring prestress, and ignoring any bars in compression.

Once the level of centroid of tension steel is found, the width term b_t needs to be calculated. This includes interrogation of all the section coordinates (as for x and h). If more than 2 sides cross the level of centroid of tension steel, the width is taken as the distance between the two extreme dimensions. This needs to be checked for sections with large voids, or channels with thin legs, as the term $b_t \times h$ is used to make an approximation for the force in the tension zone and assess the area of tension steel versus the area of tension concrete. It may be appropriate to substitute a smaller value of b_t.

BS5400 includes a notional surface a distance 'c_{nom}' from the bars. AdSec will look at all bars to define this surface excluding any with 'negative cover'. This should be reviewed, particular for sections with sharp acute angles and re-entrant corners. In the example below the adjustment to sides A and side C may not be the adjustment that would be chosen by engineering judgement. In this situation, the cracking parameters output for the relevant sides can be extracted from the output, and the results recalculated, substituting the corrected values.



The crack width equations in BS5400-4 are either equation 24

$$w = \frac{3a_{cr}\varepsilon_m}{1+2\left(a_{cr}-c_{nom}\right)/\left(h-d_c\right)}$$

where the strain ε_m is given by equation 25

$$\varepsilon_m = \varepsilon_1 - \left[\frac{3.8b_t h\left(a'-d_c\right)}{\varepsilon_s A_s\left(h-d_c\right)}\right]\left[\left(1-\frac{M_q}{M_g}\right)\times 10^{-9}\right]$$

or the alternative equation 26

$$w = 3a_{cr}\varepsilon_m$$

are offered by AdSec

Concrete material models

Symbols

f	concrete stress
f_c	concrete strength
ε	Concrete strain
ε_c	Strain at which concrete stress is maximum
ε_{cu}	Strain at which concrete fails

Units

The default units are:

Stress, strength	MPa (psi)
Elastic modulus	GPa (psi)

Concrete material models for different codes

Different material models are available for different design codes. These are summarised below:

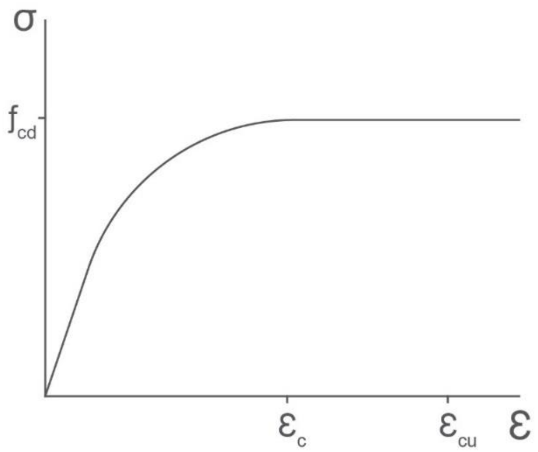
	ACI 318	AS 3600	BS 5400	BS 8110	CSA A23.3	CSA S6	EN 1992	HK CP	HK SDM	IRC:112	IRS Bridge	IS 456
Compression												
Parabola-rectangle	•	•	•	•	•	•	•	•	•	•	•	•
Rectangle	•	•		•	•	•	•	•		•		•
Bilinear							•			•		
Linear	•	•	•	•	•	•	•	•	•	•	•	•
FIB				•			•	•		•		•
Popovics	•	•			•	•						

EC2 Confined							●			●		
AISC 360 filled tube	●											
Explicit	●	●	●	●	●	●	●	●	●	●	●	●
Tension												
No-tension	●	●	●	●	●	●	●	●	●	●	●	●
Linear	●	●		●	●	●	●	●		●		●
Interpolated	●	●			●	●	●			●		
BS8110 - 2				●				●				●
TR 59				●				●				●
PD 6687							●					
Explicit	●	●	●	●	●	●	●	●	●	●	●	●
Explicit envelope	●	●	●	●	●	●	●	●	●	●	●	●

- inferred from rectangular block
- PD 6687 variant of EN 1992 only

Parabola-rectangle

Parabola-rectangles are commonly uses for concrete stress-strain curves.



Parabola-rectangle

The parabolic curve can be characterised as

$$\frac{f}{f_{cd}} = a\left(\frac{\varepsilon}{\varepsilon_c}\right)^2 + b\left(\frac{\varepsilon}{\varepsilon_c}\right)$$

At strains above ε_c the stress remains constant. For most design codes the parabola is taken as having zero slope where it meets the horizontal portion of the stress-strain curve.

$$\frac{f}{f_{cd}} = \left[1-\left(1-\frac{\varepsilon}{\varepsilon_c}\right)^2\right]$$

The Hong Kong Code of Practice (supported by the Hong Kong Institution of Engineers) interpret the curve so that the initial slope is the elastic modulus (meaning that the parabola is not tangent to the horizontal portion of the curve).

$$\frac{f}{f_{cd}} = \left[1-\left(\frac{E}{E_s}\right)\right]\left(\frac{\varepsilon}{\varepsilon_c}\right)^2 + \left(\frac{E}{E_s}\right)\left(\frac{\varepsilon}{\varepsilon_c}\right)$$

where the secant modulus is

$$E_s = \frac{f_{cd}}{\varepsilon_c}$$

In Eurocode the parabola is modified

$$\frac{f}{f_{cd}} = \left[1-\left(1-\frac{\varepsilon}{\varepsilon_c}\right)^n\right]$$

and

$$n = 2 \qquad f_c \le 50\text{MPa}$$

$$n = 1.4 + 23.4[(90 - f_c)/100]^4 \qquad f_c > 50\text{MPa}$$

EC2 Confined

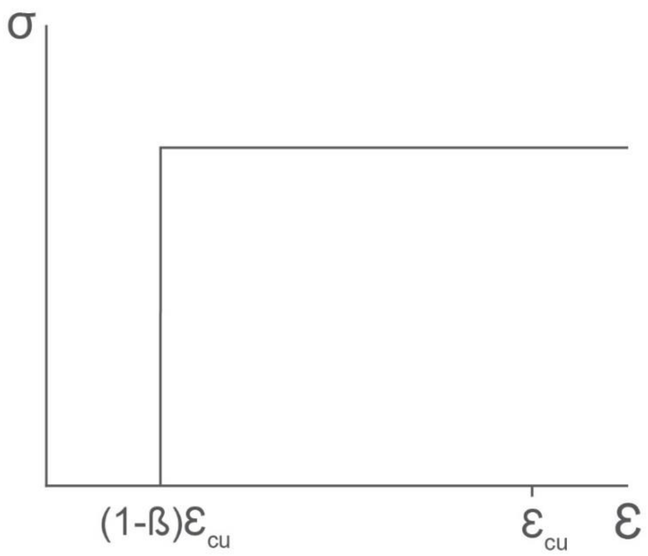
The EC2 confined model is a variant on the parabola-rectangle. In this case the confining stress σ increases the compressive strength and the plateau and failure strains.

$$f_{c,c} = \begin{cases} f_c(1 + 5\sigma/f_c) & \sigma \le 0.05 f_c \\ f_c(1.125 + 2.5\sigma/f_c) & \sigma > 0.05 f_c \end{cases}$$

$$\varepsilon_{c,c} = \varepsilon_c (f_{c,c}/f_c)^2$$
$$\varepsilon_{cu,c} = \varepsilon_{cu} + 0.2\sigma/f_c$$

Rectangle

The rectangular stress block has zero stress up to a strain of ε_c (controlled by β) and then a constant stress of αf_{cd}.



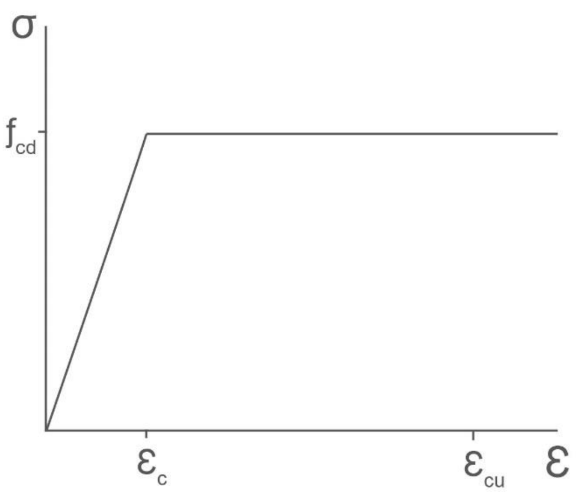
Rectangular

		α	β	
ACI 318		1	0.85 - 0.05(f_c - 30)/7	[0.65:0.85]
AS3600	2001	1	0.85 - 0.07(f_c - 28)	[0.65:0.85]
AS3600	2009	1	1.05 - 0.007f_c	[0.67:0.85]

BS5400		0.6/0.67	1
BS8110		1	0.9
CSA A23.3		1	max(0.67, 0.97 - 0.0025 f_c)
CSA S6		1	max(0.67, 0.97 - 0.0025 f_c)
EN 1992		1 $f_c \leq$ 50MPa 1 - (f_c - 50)/200 $f_c >$ 50MPa	0.8 $f_c \leq$ 50MPa 0.8 - (f_c - 50)/400 $f_c >$ 50MPa
HK CP	> 2004	1	0.9
HK CP	2007 >	1	0.9 $f_c \leq$ 45MPa 0.8 $f_c \leq$ 70MPa 0.72 $f_c \leq$ 100MPa
HK SDM		0.6/0.67	1
IRC:112		1 $f_c \leq$ 60MPa 1 - (f_c - 60)/250 $f_c >$ 60MPa	0.8 $f_c \leq$ 60MPa 0.8 - (f_c - 60)/500 $f_c >$ 60MPa
IRS Bridge		0.6/0.67	1
IS 456		0.8	0.84

Bilinear

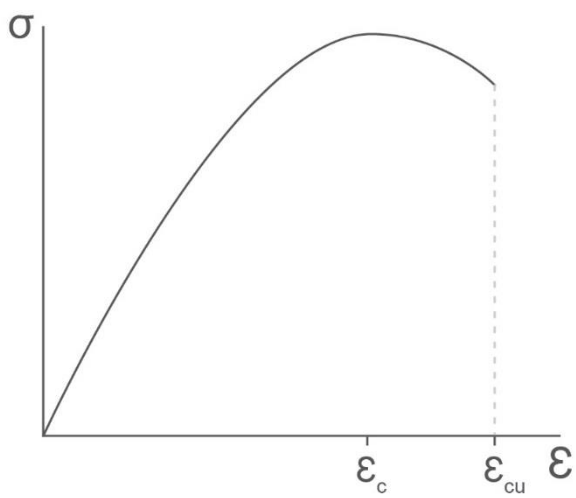
The bilinear curve is linear to the point (ε_c, f_{cd}) and then constant to failure.



Bilinear

FIB

The FIB model code defines a schematic stress-strain curve. This is used in BS 8110-2, EN1992-1 and IRC:112.



FIB Schematic

This has a peak stress f_{cFIB}

This is defined as

$$\frac{f}{f_{cFIB}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta}$$

with

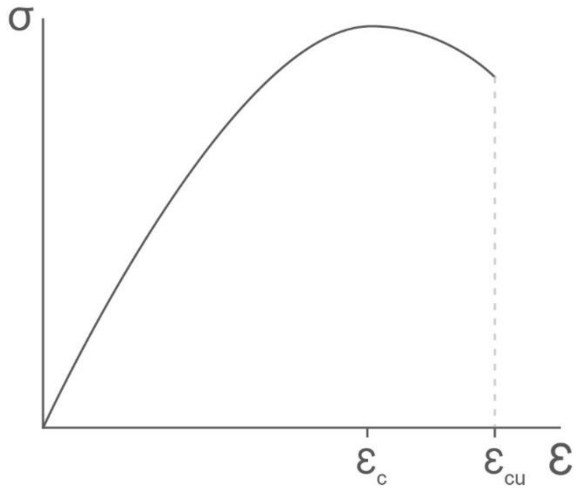
$$k = \alpha \frac{E_c}{f_c / \varepsilon_c}$$

Where the factor α is code dependent.

Code	f_{cFIB}	α
BS 8110-2	$0.8 f_c$	1.4
EN 1992-1	$f_c + 8MPa$	1.05
IRC:112	$f_c + 10MPa$	1.05

Popovics

There are a series of curves based on the work of Popovics.



Popovics

These have been adjusted and are based on the Thorenfeldt base curve.

In the Canadian offshore code (CAN/CSA S474-04) this is characterised by

$$\frac{f}{f_c} = k_3 \eta \frac{n}{n-1+\eta^{nk}}$$

with (in MPa)

$$\eta = \frac{\varepsilon}{\varepsilon_c}$$

$$k_3 = 0.6 + \frac{10}{f_c}$$

$$n = 0.8 + \frac{f_c}{17}$$

$$\varepsilon_c = \frac{f_c}{E_c}\frac{n}{n-1}$$

$$k = \begin{cases} 1 & \eta \le 1 \\ 0.67 + \dfrac{f_c}{62} & \eta > 1 \end{cases}$$

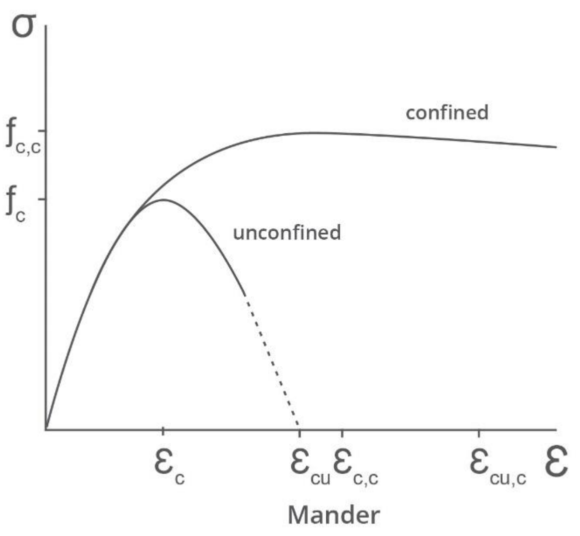
The peak strain is referred to elsewhere as ε_{pop}.

$$\varepsilon_{pop} = \varepsilon_c$$

All the concrete models require a strength value and a pair of strains: the strain at peak stress or transition strain and the failure strain.

Mander & Mander confined curve

The Mander[1] curve is available for both strength and serviceability analysis and the Mander confined curve for strength analysis.



For unconfined concrete, the peak of the stress-strain curve occurs at a stress equal to the unconfined cylinder strength f_c and strain ε_c generally taken to be 0.002. Curve constants are calculated from

$$E_{\sec} = f_c / \varepsilon_c$$

and

$$r = \frac{E}{E - E_{\sec}}$$

Then for strains $0 \le \varepsilon \le 2\varepsilon_c$ the stress σ can be calculated from:

$$\sigma = f_c \frac{\eta\, r}{r - 1 + \eta^r}$$

where

$$\eta = \frac{\varepsilon}{\varepsilon_c}$$

[1] Mander J, Priestly M, and Park R. Theoretical stress-strain model for confined concrete. Journal of Structural Engineering, 114(8), pp1804-1826, 1988.

The curve falls linearly from $2\varepsilon_c$ 2eco to the 'spalling' strain ε_{cu} . The spalling strain can be taken as 0.005-0.006.

To generate the confined curve the confined strength $f_{c,c}$ must first be calculated. This will depend on the level of confinement that can be achieved by the reinforcement. The maximum strain $\varepsilon_{cu,c}$ also needs to be estimated. This is an iterative calculation, limited by hoop rupture, with possible values ranging from 0.01 to 0.06. An estimate of the strain could be made from EC2 formula (3.27) above with an upper limit of 0.01.

The peak strain for the confined curve $\varepsilon_{c,c}$ is given by:

$$\varepsilon_{c,c} = \varepsilon_c\left[1+5\left(\frac{f_{c,c}}{f_c}-1\right)\right]$$

Curve constants are calculated from

$$E_{\text{sec}} = f_{c,c}/\varepsilon_{c,c}$$

and

$$r = \frac{E}{E - E_{\text{sec}}}$$

as before.

E is the tangent modulus of the unconfined curve, given above.

Then for strains $0 \le \varepsilon \le \varepsilon_{cu,c}$ the stress σ can be calculated from:

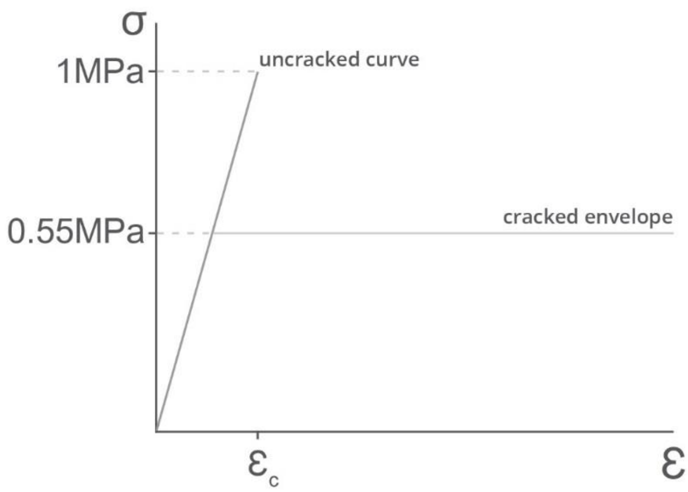
$$\sigma = f_{c,c}\frac{\eta\, r}{r-1+\eta^r}$$

where

$$\eta = \frac{\varepsilon}{\varepsilon_{c,c}}$$

BS8110-2 tension curve

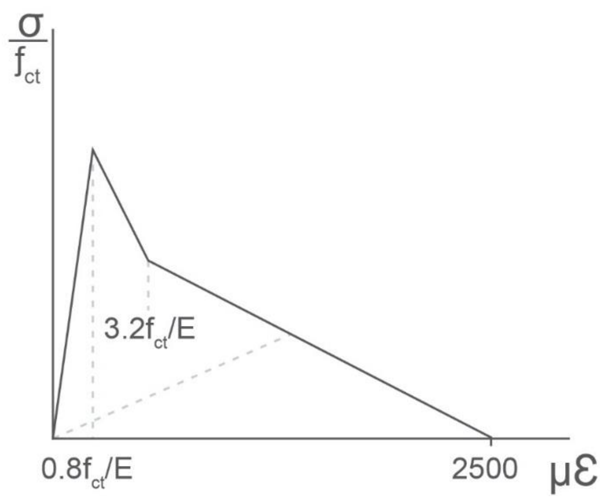
BS8110-2 define a tension curve for serviceability



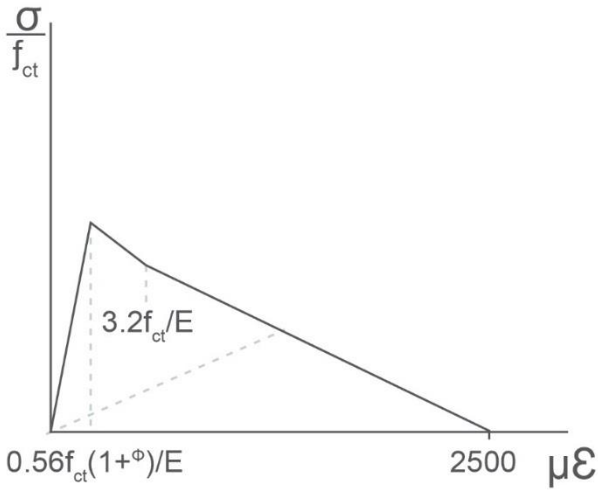
BS8110-2

TR59

Technical report 59 defines an envelope for use with concrete in tension for serviceability. The material is assumed to behave in a linearly elastic manner, with the elastic modulus reduced beyond the peak stress/strain point based on the envelope in the figures below



Tension stiffening for short-term loads



Tension stiffening for long-term loads

Interpolated

Interpolated strain plains to ACI318 and similar codes

ACI318 and several other codes give a method to compute a value of the second moment of area intermediate between that of the uncracked, I_g, and fully cracked, I_{cr}, values, using the following expression:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

where M_{cr} is the cracking moment and M_a is the applied moment.

AdSec SLS analyses determine a strain plane intermediate to the uncracked and fully cracked strain planes. The program determines a value for ζ, the proportion of the fully cracked strain plane to add to the proportion $(1-\zeta)$ of the uncracked plane so that the resulting plane is compatible with ACI318's approach. Unfortunately, since ACI318's expression is an interpolation of the inverse of the curvatures, rather than the curvatures themselves, there is no direct conversion. It should also be noted that although I_g is defined as the value of second moment of area ignoring the reinforcement, it is assumed that this definition was made for simplicity, and AdSec includes the reinforcement.

Let $\alpha = (M_{cr}/M_a)^3$, the uncracked curvature be κ_I and the fully cracked curvature be κ_{II}.

To ACI318, the interpolated curvature

$$\kappa = \frac{1}{[\alpha/\kappa_I + (1-\alpha)/\kappa_{II}]},$$

and the aim is to make this equivalent to

$$\kappa = \zeta\kappa_{II} + (1-\zeta)\kappa_I,$$

Equating these two expressions gives

$$\alpha\zeta\,\kappa_{II}/\kappa_I + \alpha(1-\zeta) + (1-\alpha)\zeta + (1-\alpha)(1-\zeta)\kappa_I/\kappa_{II} = 1$$

which can be re-arranged to give

$$\zeta = \frac{1}{[1+(\kappa_{II}/\kappa_I)/(1/\alpha-1)]}$$

The ratio κ_{II}/κ_I is appropriate for uniaxial bending. For applied loads (N, M_y, M_z), and uncracked and fully cracked strain planes $(\varepsilon_I, \kappa_{yI}, \kappa_{zI})$ and $(\varepsilon_{II}, \kappa_{yII}, \kappa_{zII})$ respectively, κ_{II}/κ_I is replaced by the ratio $(N\varepsilon_{II} + M_y\kappa_{yII} + +M_z\kappa_{zII})/(N\varepsilon_I + M_y\kappa_{yI} + +M_z\kappa_{zI})$, which is independent of the location chosen for the reference point. In the absence of axial loads, this ratio ensures that the curvature about the same axis as the applied moment will comply with ACI318; in the absence of moments, the axial strain will follow a relationship equivalent to that in ACI318 but using axial stiffness as imposed to flexural stiffness.

The ratio (M_{cr}/M_a) is also inappropriate for general loading. For the general case, it is replaced by the ratio (f_{ct}/σ_{tI}), where f_{ct} is the tensile strength of the concrete and σ_{tI} is the maximum concrete tensile stress on the uncracked section under applied loads.

Summary: $$\zeta = 1\Bigg/\left[1+\frac{(N\varepsilon_{II} + M_y\kappa_{yII} + M_z\kappa_{zII})/(N\varepsilon_I + M_y\kappa_{yI} + M_z\kappa_{zI})}{(\sigma_{tI}/f_{ct})^3 - 1}\right]$$

Since ζ is larger for short-term loading, all curvatures and strains are calculated based on short-term properties regardless of whether ζ is subsequently used in a long-term serviceability calculation.

Concrete properties

Notation

f_c	concrete strength
f_{cd}	concrete design strength
f_{ct}	concrete tensile strength
E	elastic modulus
ν	Poisson's ratio (0.2)
α	coefficient of thermal expansion (varies but 1×10^{-6}/°C assumed)
ε_{cu}	strain at failure (ULS)

ε_{ax} compressive strain at failure (ULS)

ε_{plas} strain at which maximum stress is reached (ULS)

$\varepsilon_{\max}$ assumed maximum strain (SLS)

ε_{peak} strain corresponding to (first) peak stress (SLS)

ε_{pop} strain corresponding to peak stress in Popovics curve (SLS)

ε_{β} $\varepsilon_{\beta} = (1-\beta)\varepsilon_u$

ACI

The density of normal weight concrete is assumed to be 2200kg/m^3.

The design strength is given in 22.2.2.4.1 by

$$f_{cd} = 0.85 f_c$$

The tensile strength is given in Equation 9-10 by

$$f_{ct} = 0.62\sqrt{f_c}$$

$$f_{ct} = 7.5\sqrt{f_c} \quad \text{(US units)}$$

The elastic modulus is given in 8.5.1 as

$$E = 4.7\sqrt{f_c}$$

$$E = 57000\sqrt{f_c} \quad \text{(US units)}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.003	ε_{cu}	$(1 - 3\beta)\ \varepsilon_{cu}$		
Rectangle	0.003	ε_{cu}	ε_{β}		
Bilinear	0.003	ε_{cu}	$(1 - 2\beta)\ \varepsilon_{cu}$		
Linear				0.003	ε_{max}
FIB					

Popovics				0.003	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.003	ε_{cu}		0.003	

AS

The density of normal weight concrete is taken as 2400kg/m^3(3.1.3).

The design strength is given in 10.6.2.5(b) by

$$f_{cd} = \alpha_2 f_c$$

with

$$\alpha_2 = 1 - 0.003 f_c$$

and limits of [0.67:0.85]

The tensile strength is given in 3.1.1.3 by

$$f_{ct} = 0.6\sqrt{f_c}$$

The elastic modulus is given (in MPa) in 3.1.2 as

$$E = \rho^{1.5} \times 0.043\sqrt{f_{cmi}} \qquad f_{cmi} \le 40MPa$$

$$E = \rho^{1.5} \times 0.024\sqrt{f_{cmi}} + 0.1 \qquad f_{cmi} > 40MPa$$

This tabulated in Table 3.1.2.

f_c (MPa)	E (GPa)
20	24.0
25	26.7
32	30.1
40	32.8
50	34.8
65	37.4
80	39.6

100	42.2

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle					
Rectangle	0.003	0.0025	ε_{β}	0.003	ε_{β}
Bilinear					
Linear				0.003	ε_{max}
FIB					
Popovics				ε_{max}	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.003	0.0025		0.003	

BS 5400

The density of normal weight concrete is given in Appendix B as 2300kg/m^3.

The design strength is given in Figure 6.1 by

$$f_{cd} = 0.6 f_c / \gamma$$

The tensile strength is given in 6.3.4.2 as

$$f_{ct} = 0.36\sqrt{f_c}$$

but A.2.2 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus tabulated in 4.3.2.1 Table 3

f_c (MPa)	E (GPa)

20	25.0
25	26.0
32	28.0
40	31.0
50	34.0
60	36.0

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		
Rectangle					
Bilinear					
Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

BS 8110

The density of normal weight concrete is given in section 7.2 of BS 8110-2 as 2400kg/m^3.

The design strength is given in Figure 3.3 by

$$f_{cd} = 0.67 f_c / \gamma$$

The tensile strength is given in 4.3.8.4 as

$$f_{ct} = 0.36\sqrt{f_c}$$

but Figure 3.1 in BS 8110-2 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is given in Equation 17

$$E = 20 + 0.2 f_c$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_u	ε_{cu}	ε_{RP}	0.0035*	ε_{RP}
Rectangle	ε_u	ε_{cu}	ε_β		
Bilinear					
Linear				ε_u	ε_{max}
FIB				ε_u	0.0022
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	ε_u	ε_{cu}		ε_u	

$$\varepsilon_u = \begin{cases} 0.0035 & f_c \le 60MPa \\ 0.0035 - 0.001 \times \dfrac{(f_c - 60)}{50} & \end{cases}$$

$$\varepsilon_{RP} = 2.4 \times 10^{-4}\sqrt{\frac{f_c}{\gamma}}$$

CSA A23.3 / CSA S6

The density of normal weight concrete is assumed to be 2300 kg/m^3; see 8.6.2.2 (A23.3) and 8.4.1.7 (S6).

The design strength is given in 10.1.7 by

$$f_{cd} = \max(0.67, 0.85 - 0.0015 f_c)\varphi f_c$$

The tensile strength is given in Equation 8.3 (A23.3) and 8.4.1.8.1 in (S6)

$$f_{ct} = 0.6\sqrt{f_c} \quad \text{(for CSA A23.3)}$$

$$f_{ct} = 0.4\sqrt{f_c} \quad \text{(for CSA S6)}$$

For normal weight concrete the modulus is given in A23.3 Equation 8.2.

$$E = 4.5\sqrt{f_c}$$

and in CSA S6 8.4.1.7

$$E = 3.0\sqrt{f_c} + 6.9$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	$(1 - 3\beta)\ \varepsilon_u$		
Rectangle	0.0035	ε_{cu}	ε_β		
Bilinear	0.0035	ε_{cu}	$(1 - 2\beta)\ \varepsilon_u$		
Linear				0.0035	ε_{max}
FIB					
Popovics				0.0035	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

EN 1992

The density of normal weight concrete is specified in 11.3.2 as 2200 kg/m^3.

The design strength is given in 3.1.6 by

$$f_{cd} = \alpha_{cc} f_c / \gamma$$

For the rectangular stress block this is modified to

$$f_{cd} = \alpha_{cc} f_c / \gamma \qquad f_c \le 50MPa$$

$$f_{cd} = \alpha_{cc} 1.25(1 - f_c/250) f_c / \gamma \qquad f_c > 50MPa$$

The tensile strength is given in Table 3.1 as

$$f_{ct} = 0.3 f_c^{2/3} \qquad f_c \le 50MPa$$

$$f_{ct} = 2.12 \ln(1 + (f_c + 8)/10) \qquad f_c > 50MPa$$

The modulus is defined in Table 3.1

$$E = 22\left(\frac{f_c + 8}{10}\right)^{0.3}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_{cu2}	ε_{c2}	ε_{c2}		
Rectangle	ε_{cu3}	ε_{c3}	ε_{β}		
Bilinear	ε_{cu3}	ε_{c3}	ε_{c3}	ε_{cu3}	ε_{c3}
Linear				ε_{cu2}	ε_{c2}
FIB				ε_{cu1}	ε_{c1}
Popovics					
EC2 Confined	$\varepsilon_{cu2,c}$	$\varepsilon_{c2,c}$	$\varepsilon_{c2,c}$		
AISC filled tube					

Explicit	ε_{cu2}	$\varepsilon_{cu2\,?}$		ε_{cu2}	

$$\varepsilon_{c1} = 0.007 f_{cm}^{0.31} \qquad \le 0.0028$$

$$\varepsilon_{cu1} = \begin{cases} 0.0035 & f_c \le 50MPa \\ 0.0028 + 0.027\left(\dfrac{90 - f_c}{100}\right)^4 & \end{cases}$$

$$\varepsilon_{c2} = \begin{cases} 0.002 & f_c \le 50MPa \\ 0.002 + 0.000085(f_{ck} - 50)^{0.53} & \end{cases}$$

$$\varepsilon_{cu2} = \begin{cases} 0.0035 & f_c \le 50MPa \\ 0.0026 + 0.035\left(\dfrac{90 - f_c}{100}\right)^4 & \end{cases}$$

$$\varepsilon_{c3} = \begin{cases} 0.00175 & f_c \le 50MPa \\ 0.00175 + 0.00055\left(\dfrac{f_{ck} - 50}{40}\right) & \end{cases}$$

$$\varepsilon_{cu3} = \begin{cases} 0.0035 & f_c \le 50MPa \\ 0.0026 + 0.035\left(\dfrac{90 - f_c}{100}\right)^4 & \end{cases}$$

HK CP

The density of normal weight concrete is assumed to be 2400kg/m^3.

The design strength is given in Figure 6.1 by

$$f_{cd} = 0.67 f_c / \gamma$$

The tensile strength is given in 12.3.8.4 as

$$f_{ct} = 0.36\sqrt{f_c}$$

but 7.3.6 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is defined in 3.1.5

$$E = 3.46\sqrt{fc} + 3.21$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}

Parabola-rectangle	ε_u	ε_{cu}	ε_{RP}		
Rectangle	ε_u	ε_{cu}	ε_β		
Bilinear					
Linear				ε_u	ε_u
FIB				ε_u	0.0022
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	ε_u	ε_{cu}		ε_u	

$$\varepsilon_u = 0.0035 - 0.00006 \times \sqrt{f_c - 60} \qquad f_c > 60MPa$$

$$E_d = 3.46\sqrt{\frac{f_c}{\gamma}} + 3.21GPa$$

$$\varepsilon_{RP} = 1.34\frac{f_c/\gamma}{E_d}$$

HK SDM

The density of normal weight concrete is assumed to be 2400kg/m^3.

The design strength is given in 5.3.2.1(b) of BS 5400-4 by

$$f_{cd} = 0.6 f_c/\gamma$$

The tensile strength is given in 6.3.4.2 as

$$f_{ct} = 0.36\sqrt{f_c}$$

but from BS5400 a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is tabulated in Table 21

f_c (MPa)	E (GPa)
20	18.9
25	20.2
32	21.7
40	24.0
45	26.0
50	27.4
55	28.8
60	30.2

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		
Rectangle					
Bilinear					
Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

IRC 112

The density of normal eight concrete is assume to be 2200kg/m^3.

The design strength is given in 6.4.2.8

$$f_{cd} = 0.67 f_c / \gamma$$

In A2.9(2) the strength is modified for the rectangular stress block to

$$f_{cd} = 0.67 f_c / \gamma \qquad f_c \le 60MPa$$

$$f_{cd} = 0.67(1.24 - f_c/250) f_c / \gamma \qquad f_c > 60MPa$$

The tensile strength is given in by A2.2(2) by

$$f_{ct} = 0.259 f_c^{2/3} \qquad f_c \le 60MPa$$

$$f_{ct} = 2.27 \ln(1 + (f_c + 10)/12.5) \qquad f_c > 60MPa$$

The elastic modulus is given in A2.3, equation A2-5

$$E = 22\left(\frac{f_c + 10}{12.5}\right)^{0.3}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_{cu2}	ε_{c2}	ε_{c2}	ε_{cu2}	ε_{c2}
Rectangle	ε_{cu3}	ε_{c3}	ε_{β}		
Bilinear	ε_{cu3}	ε_{c3}	ε_{c3}	ε_{cu3}	ε_{c3}
Linear				ε_{cu2}	ε_{c2}
FIB				ε_{cu1}	ε_{c1}
Popovics					
EC2 Confined	$\varepsilon_{cu2,c}$	$\varepsilon_{c2,c}$	$\varepsilon_{c2,c}$		
AISC filled tube					
Explicit	ε_{cu2}	ε_{cu2} ?		ε_{cu2}	

$$\varepsilon_{c1} = 0.00653(f_c + 10)^{0.31} \qquad \leq 0.0028$$

$$\varepsilon_{cu1} = \begin{cases} 0.0035 & f_c \leq 60MPa \\ 0.0028 + 0.027\left(\dfrac{90 - 0.8 f_c}{100}\right)^4 & \end{cases}$$

$$\varepsilon_{c2} = \begin{cases} 0.002 & f_c \leq 60MPa \\ 0.002 + 0.000085(0.8 f_{ck} - 50)^{0.53} & \end{cases}$$

$$\varepsilon_{cu2} = \begin{cases} 0.0035 & f_c \leq 60MPa \\ 0.0026 + 0.035\left(\dfrac{90 - 0.8 f_c}{100}\right)^4 & \end{cases}$$

$$\varepsilon_{c3} = \begin{cases} 0.00175 & f_c \leq 60MPa \\ 0.00175 + 0.00055\left(\dfrac{0.8 f_{ck} - 50}{40}\right) & \end{cases}$$

$$\varepsilon_{cu3} = \begin{cases} 0.0035 & f_c < 50MPa \\ 0.0026 + 0.035\left(\dfrac{90 - 0.8 f_c}{100}\right)^4 & \end{cases}$$

IRS Bridge

The density is assumed to be 2300kg/m^3.

The design strength is given in 15.4.2.1(b) by

$$f_{cd} = 0.6 f_c / \gamma$$

The tensile strength is given in 16.4.4.3 as

$$f_{ct} = 0.37\sqrt{f_c}$$

The elastic modulus is tabulated in 5.2.2.1

f_c (MPa)	E (GPa)
20	25.0
25	26.0
32	28.0
40	31.0
50	34.0

60 36.0

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		
Rectangle					
Bilinear					
Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

IRC 456

The density is assumed to be 2200 kg/m^3.

The design strength is given in Figure 21by

$$f_{cd} = 0.67 f_c / \gamma$$

The tensile strength is inferred from 6.2.2 as

$$f_{ct} = 0.7 \sqrt{f_c}$$

The elastic modulus is defined in 6.2.3.1

$$E = 5\sqrt{f_c}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	0.002	0.002		
Rectangle	0.0035	0.002	ε_{β}		
Bilinear					
Linear				0.0035	ε_{max}
FIB				0.0035	0.0022
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	0.002		0.0035	

Rebar material models

Symbols

f rebar stress

f_y rebar strength

f_u rebar ultimate strength

ε rebar strain

ε_p strain at which rebar stress is maximum

ε_u strain at which rebar fails

Rebar material models for different codes

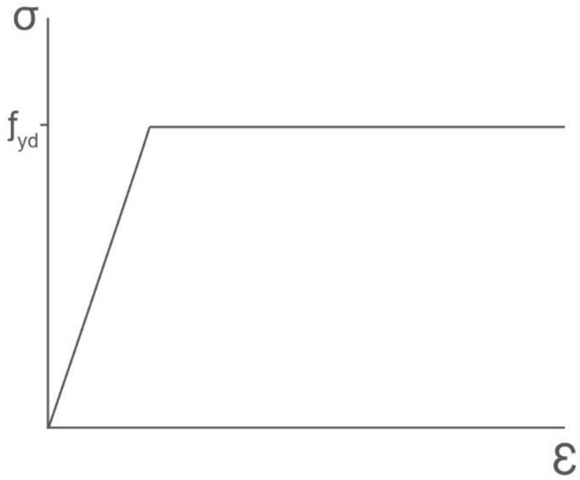
Different material models are available for different design codes. These are summarised below:

	ACI 318	AS 3600	BS 5400	BS 8110	CSA A23.3	CSA S6	EN 1992	HK CP	HK SDM	IRC:112	IRS Bridge	IS 456
Elastic-plastic	•	•		•	•	•	•	•		•	•	•
Elastic-hardening							•			•		
BS 5400			•						•			
Pre-stress			•	•					•		•	
Progressive yield											•	•
Park	•											
Linear	•	•	•	•	•	•	•	•	•	•	•	•
No-compression	•	•	•	•	•	•	•	•	•	•	•	•

ASTM strand					•	•						
Explicit	•	•	•	•	•	•	•	•	•	•	•	•

Elastic-plastic

The initial slope is defined by the elastic modulus, E. Post-yield the stress remains constant until the failure strain, ε_u, is reached.



Elastic-plastic

For some codes (CAN/CSA) the initial slope is reduced to ϕE.

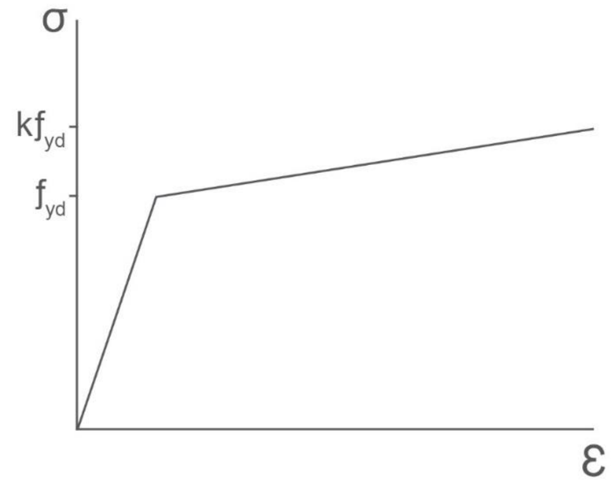
Elastic-hardening

The initial slope is defined by the elastic modulus, E, after yield the hardening modulus E_h governs as stress rises from (ε_y, f_{yd}) to (ε_u, f_u). For EN 1992 the hardening modulus is defined in terms of a hardening coefficient k and the final point is $(\varepsilon_{uk}, kf_{yd})$ where the failure strain is reduced to ε_{ud} (typically $0.9\varepsilon_{uk}$).

The relationship between hardening modulus and hardening coefficient is:

$$E_h = \frac{(k-1)f_y}{\varepsilon_{uk} - f_y/E}$$

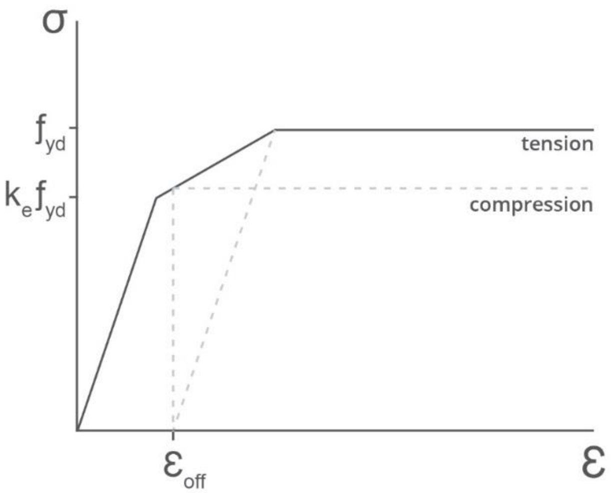
$$k = \frac{E_h(\varepsilon_{uk} - f_y/E)}{f_y} + 1$$



Elastic-hardening

The material fails at ε_{ud} where $\varepsilon_{ud} < \varepsilon_{uk}$. This is defined in Eurocode and related codes.

BS 5400



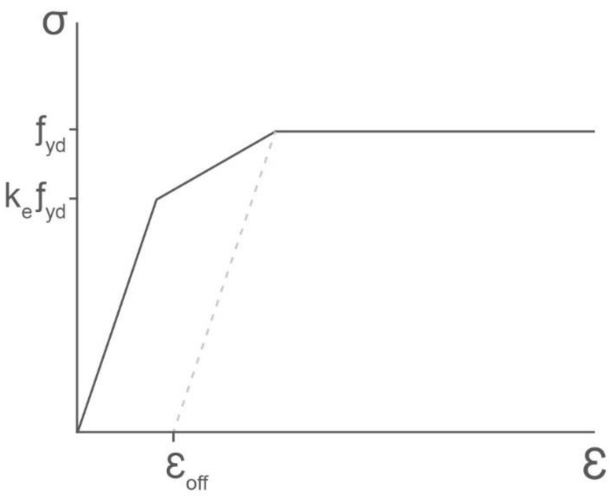
BS 5400

In tension the initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$. The slope then reduces until the material is fully plastic, f_{yd}, at $\varepsilon = \varepsilon_{off} + f_{yd}/E$. Post-yield the stress remains constant until the failure strain, ε_u, is reached. For BS5400 $k_e = 0.8$ and $\varepsilon_{off} = 0.002$.

In compression the initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$ or a strain of ε_{off}. It then follows the slope of the tension curve post-yield and when the strain reaches ε_{off} the stress remain constant until failure

Pre-stress

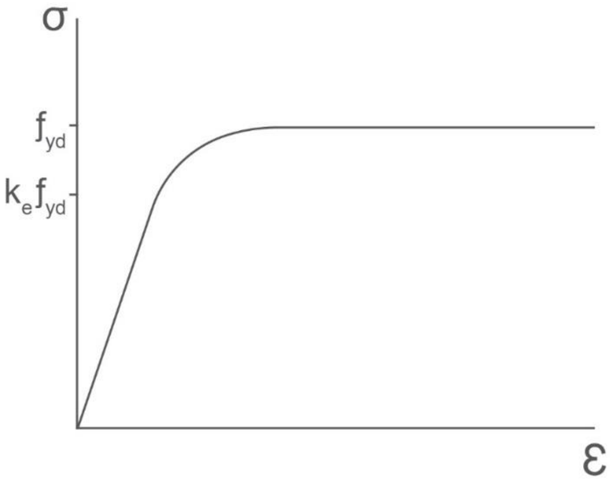
The initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$. The slope then reduces until the material is fully plastic, f_{yd}, at $\varepsilon = \varepsilon_{off} + f_{yd}/E$. Post-yield the stress remains constant until the failure strain, ε_u, is reached. For BS8110 and related codes $k_e = 0.8$ and $\varepsilon_{off} = 0.005$.



Trilinear

Progressive yield

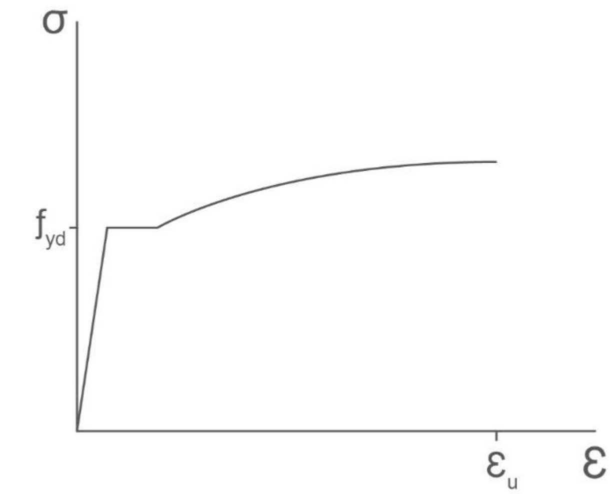
The initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$. The slope then reduces in a series of steps until the material is fully plastic, after which the stress remain constant. The points defining the progressive yield are code dependent.



Progressive yield

Park

The initial slope is defined by the elastic modulus, E, until the stress reaches f_{yd}. The slope is then zero for a short strain range, then rising to a peak stress before failure.



Park

$$\sigma = f_{ud} - \left(f_{ud} - f_{yd}\right)\left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_p}\right)^p$$

$$p = E\left(\frac{\varepsilon_u - \varepsilon_p}{f_{ud} - f_{yd}}\right)$$

Linear

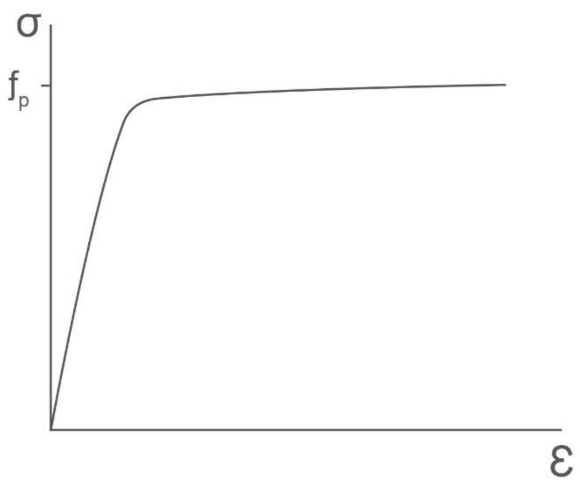
The initial slope is defined by the elastic modulus, E, until the failure strain is reached.

No-compression

This is a linear model when in tension which has no compressive strength.

ASTM strand

The ASTM A 416 defines a stress-strain curve doe seven-wire strands. This has an initial linear relationship up to a strain of 0.008 with progressive yield till failure.



ASTM Strand

The stress strain curves are defined for specific strengths.

For Grade 250[2] (1725 MPa) the stress-strain curve is defined as

$$\sigma = 197000\varepsilon \qquad \varepsilon \le 0.008$$

$$\sigma = 1710 - \frac{0.4}{\varepsilon - 0.006} \qquad \varepsilon > 0.008$$

For Grade 270 (1860 MPa) the stress-strain curve is defined as

[2] Bridge Engineering Handbook, Ed. Wah-Fah Chen, Lian Duan, CRC Press 1999

$$\sigma = 197000\varepsilon \qquad \varepsilon \le 0.008$$

$$\sigma = 1848 - \frac{0.517}{\varepsilon - 0.003} \qquad \varepsilon > 0.008$$

In the Commentary to the Canadian Bridge [3] code a similar stress-strain relationship is defined.

For Grade 1749 strand

$$\sigma = E_p\varepsilon \qquad \varepsilon \le 0.008$$

$$\sigma = 1749 - \frac{0.433}{\varepsilon - 0.00614} \qquad \varepsilon > 0.008$$

For Grade 1860 strand

$$\sigma = E_p\varepsilon \qquad \varepsilon \le 0.008$$

$$\sigma = 1848 - \frac{0.517}{\varepsilon - 0.0065} \qquad \varepsilon > 0.008$$

A more detailed discussion of modelling strands can be found in the paper[4] by Devalapura and Tadros

Crack calculation

CSA S6

Code Approach

The equation for the crack width is given in section 8.12.3.2 as

$$w = k_b\beta_c s_{rm}\varepsilon_{sm}$$

The parameters k_b and β_c depend on the section and the cause of cracking. The code defines s_{rm} in mm as

$$s_{rm} = 50 + 0.25k_c\frac{d_b}{\rho_c}$$

[3] Commentary on CSA S6-14, Canadian Highway Bridge Design Code, CSA Group, 2014
[4] Stress-Strain Modeling of 270 ksi Low-Relaxation Prestressing Strands, Devalapura R K & Tadros M K, PCI Journal, March April 1992

Where

$$\rho_c = \frac{As}{A_{ct}}$$

and A_{ct} is the concrete area excluding the reinforcement. The code gives values for k_c as 0.5 for bending and 1.0 for pure tension. In AdSec we interpolate between these values using

$$k_c = \frac{(\varepsilon_{\max} + \max(\varepsilon_{\min}, 0))}{2 \times \max(|\varepsilon_{\max}|, |\varepsilon_{\min}|)}$$

But limited to the range [0.5:1].

The d_b term is taken as the average bar diameter in the tension zone, and the area of steel in ρ_c is taken as the weighted area of the bar in the tension zone

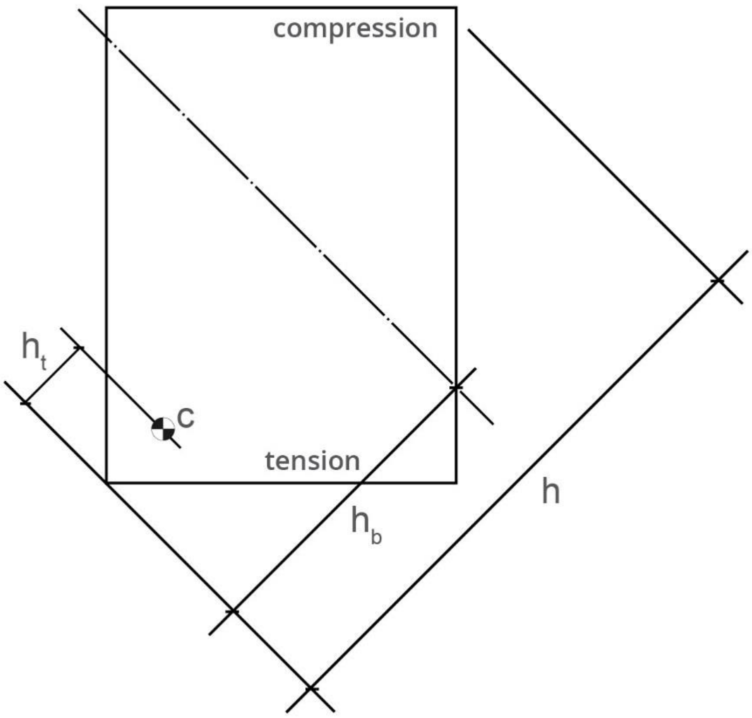
$$A_s = \sum \alpha_i \frac{\pi d_i^{\,2}}{4}$$

And the weighing is based on the stress in the bar compared with the stress in the extreme bar.

$$\alpha = \frac{\sigma_i}{\sigma_{extreme}}$$

The area of concrete in ρ_c is

$$A_c = \min(A_c(2.5h_t), A_c(h_b/3))$$



where c is the centroid of reinforcement in tension. The strain term is given in the code as

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left[1-\left(\frac{f_w}{f_s}\right)^2\right]$$

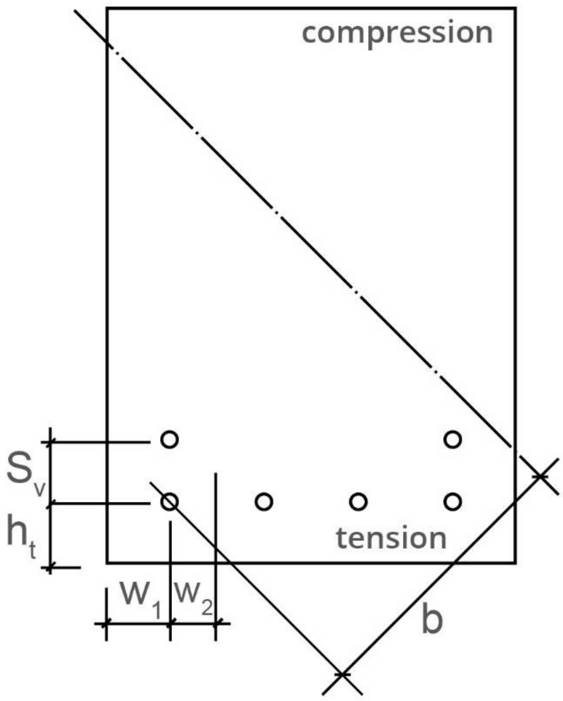
Where f_s is the stress in the reinforcement at the serviceability limit state and f_w is the stress in the reinforcement at initial cracking. In AdSec this is implemented as

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s}\left[1-\left(\frac{f_{c,rup}}{\sigma_{ct}}\right)^2\right]$$

Where σ_s is the stress in the most tensile reinforcement at the serviceability for a fully cracked calculation, $f_{c,rup}$ is the rupture strength of the concrete and σ_{ct} is the maximum tensile stress in the concrete at the extreme fibre assuming an uncracked material.

Local Approach

The above approach becomes difficult to justify when the section is not in uniaxial bending. In these cases the alternative 'local' approach can be used. This assumes there is a local relationship between a bar and the surrounding concrete.



The first stage is to identify the most tensile bar and determine the cover c to this bar. We then define h_t as the cover plus half the bar diameter.

$$h_t = c + \frac{d_b}{2}$$

The depth from the neutral axis the most tensile bar b is calculated, and h_b is then defined as

$$h_b = b + c + \frac{d_b}{2}$$

Then the concrete area is based on a dimension

$$h_c = \min\left(2.5h_t, h_b/3, \left(h_t + s_v/2\right)\right)$$

The width associated with this is

$$w_c = \min\left(5h_t, \left(w_1 + w_2\right)\right)$$

So that

$$A_c = h_c \times w_c$$

EN1992-1

The equation for the crack width is equation 7.8

$$w_k = s_{r,\max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

In this $s_{r,\max}$ is given by

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$$

The code gives values for k_1 as 0.8 for high bond bars and 1.6 for plain bars or pre-stressing tendon. The code gives values for k_2 as 0.5 for bending and 1.0 for pure tension. In AdSec we interpolate between these values using

$$k_c = \frac{\left(\varepsilon_{\max} + \max\left(\varepsilon_{\min}, 0\right)\right)}{2 \times \max\left(\left|\varepsilon_{\max}\right|, \left|\varepsilon_{\min}\right|\right)}$$

But limited to the range [0.5:1]. k_3 and k_4 are nationally determined parameters which default to 3.4 and 0.425 respectively.

Where the spacing of bar is large, $> 5\left(c + \phi/2\right)$, then

$$s_{r,\max} = 1.3\left(h - x\right)$$

$\rho_{p,eff}$ is the ratio of reinforcement to concrete in the cracking zone where the area of concrete is defined as

$$A_{c,eff} = \min\left(A_c\left(2.5\left(h-d\right), A_c\left(\left(h-x\right)/3\right), Ac\left(h/2\right)\right)\right)$$

Appendix

Alternative stress blocks

General stress blocks

Parabola-rectangles are commonly uses for concrete stress-strain curves. The parabolic curve can be characterised as

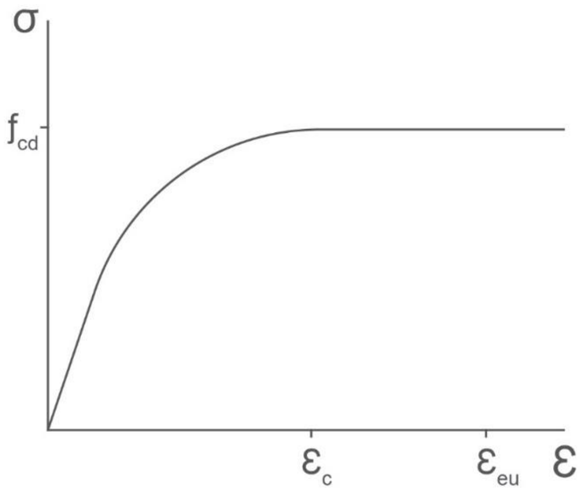
$$\frac{f}{f_c} = a\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 + b\left(\frac{\varepsilon}{\varepsilon_p}\right)$$

Define

$$f' = \frac{f}{f_c}$$

&

$$\eta = \frac{\varepsilon}{\varepsilon_p}$$



Parabola-rectangle

If the curve is taken to be tangent to the plateau then at $\eta = 1,\ \ f' = 1$ and $\dfrac{df'}{d\eta} = 0$

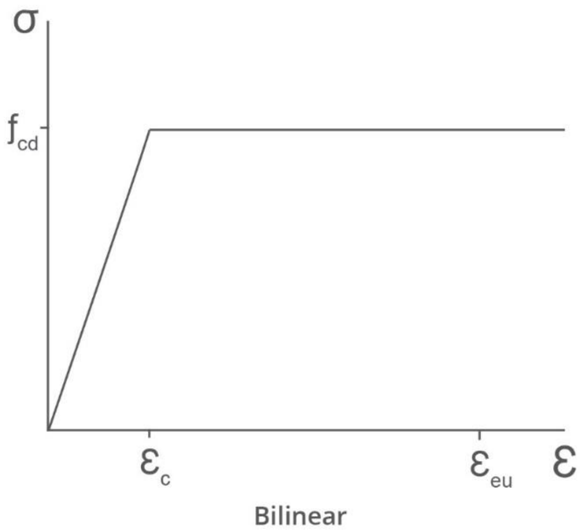
Solving for the coefficients gives $a = -1$ and $b = 2$ so

$$f' = 2\eta - \eta^2$$

The area under the curve is given by

$$A_p = \int_0^1 f' d\eta = \left[\eta^2 - \frac{\eta^3}{3}\right]_0^1 = \frac{2}{3}$$

For bi-linear curve with the strain transition at ε_b the area under the curve to ε_p is



$$A_b = \frac{\eta_b}{2} + (1-\eta_b) = 1 - \frac{\eta_b}{2}$$

Equating the areas

$$1 - \frac{\eta_b}{2} = \frac{2}{3}$$

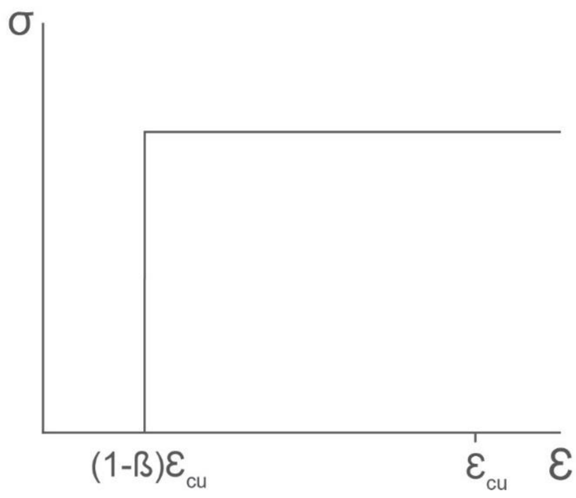
or

$$\eta_b = \frac{2}{3}$$

so

$$\varepsilon_{c,b} = \frac{2}{3}\varepsilon_{c,p}$$

For a rectangular stress block with the strain transition at ε_r the area under the curve to ε_p is



Rectangular

$$A_b = 1 - \eta_r$$

Equating the areas

$$1 - \eta_r = \frac{2}{3}$$

or

$$\eta_r = \frac{1}{3}$$

so

$$\varepsilon_{c,r} = \frac{1}{3}\varepsilon_{c,p}$$

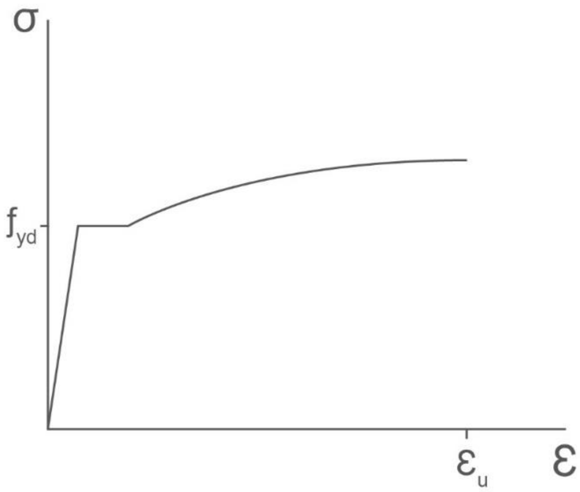
References

Popvics S (1973) A numerical approach to the complete stress-strain curve of concrete, Cement and Concrete Research 3(5) pp 553-599

Thorenfeldt E, Tomaszewicz A and Jensen J J, (1987) *Mechanical Properties of high-strength concrete and application in design*, Proceedings of the symposium on Utilization of High-Strength Concrete, Stavanger, pp 149-159

Kent, D.C., and Park, R. (1971). "Flexural members with confined concrete." Journal of the Structural Division, Proc. of the American Society of Civil Engineers, 97(ST7), 1969-1990.

Steel materials curve



Park

The steel stress-strain curve is characterised a liner response to yield, followed by a fully plastic zone, before hardening until failure.

The hardening zone can be approximated by a parabola

$$\frac{\sigma}{f_{yd}} = a\varepsilon^2 + b\varepsilon + c$$

Defining the perfectly plastic strain limit as ε_p and assuming zero slope at ε_u then

$$1 = a\varepsilon_p^2 + b\varepsilon_p + c$$

$$\frac{f_{ud}}{f_{yd}} = a\varepsilon_u^2 + b\varepsilon_u + c$$

$$b = -2a\varepsilon_u$$

The difference between the first two gives

$$\frac{f_{ud}}{f_{yd}} - 1 = a\left(\varepsilon_u^2 - \varepsilon_p^2\right) + b\left(\varepsilon_u - \varepsilon_p\right)$$

And substituting the third into this gives

$$\frac{f_{ud}}{f_{yd}} - 1 = a\left[\left(\varepsilon_u^2 - \varepsilon_p^2\right) - 2\varepsilon_u\left(\varepsilon_u - \varepsilon_p\right)\right]$$

or

$$a = \frac{1 - \left(f_{ud} / f_{yd} \right)}{\left(\varepsilon_u - \varepsilon_p \right)^2}$$

$$b = -2\varepsilon_u a$$

$$c = 1 - b\varepsilon_p - a\varepsilon_p^{\ 2}$$

