7.1 Multi-linear Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresis01.fea

Figure 7.1.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of multi-linear model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

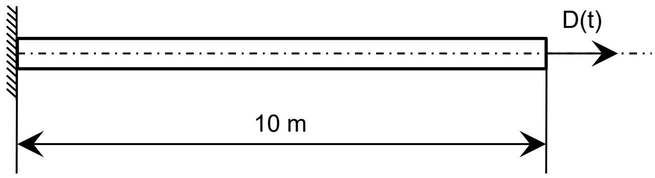


Figure 7.1.1
bar element model

Material data	Initial stiffness	$K = 4.0 \times 10^9\ N$
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.1.2
Force-Stiffness reduction factor	1st Yield point	Force : $4.0 \times 10^5\ N$ Stiffness reduction : 0.1875
	2nd Yield point	Force : $5.5 \times 10^5\ N$ Stiffness reduction : 0.041667
Hinge location	Lumped plasticity	End node

Figure 7.1.2
Prescribed displacement at right end node

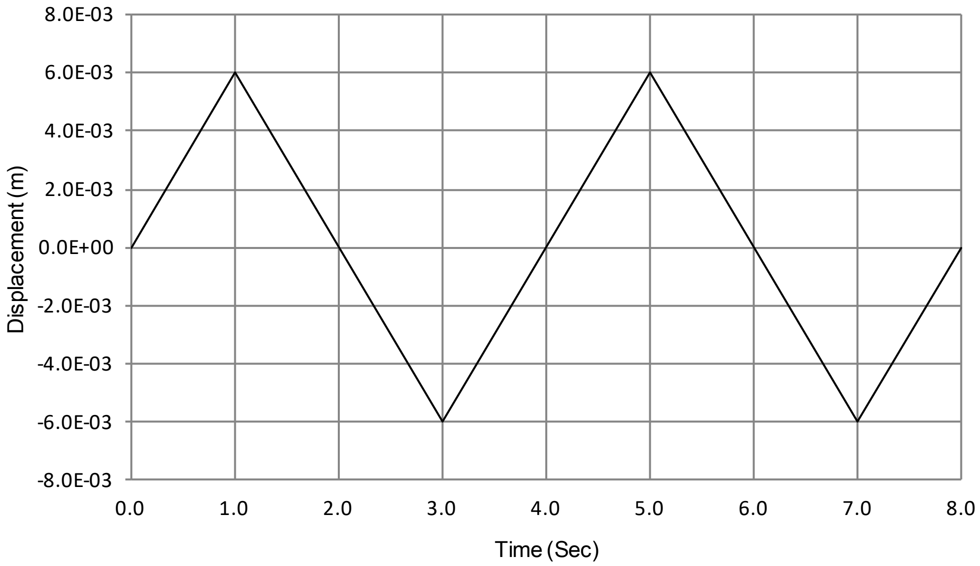


Figure 7.1.3
Force-displacement curve

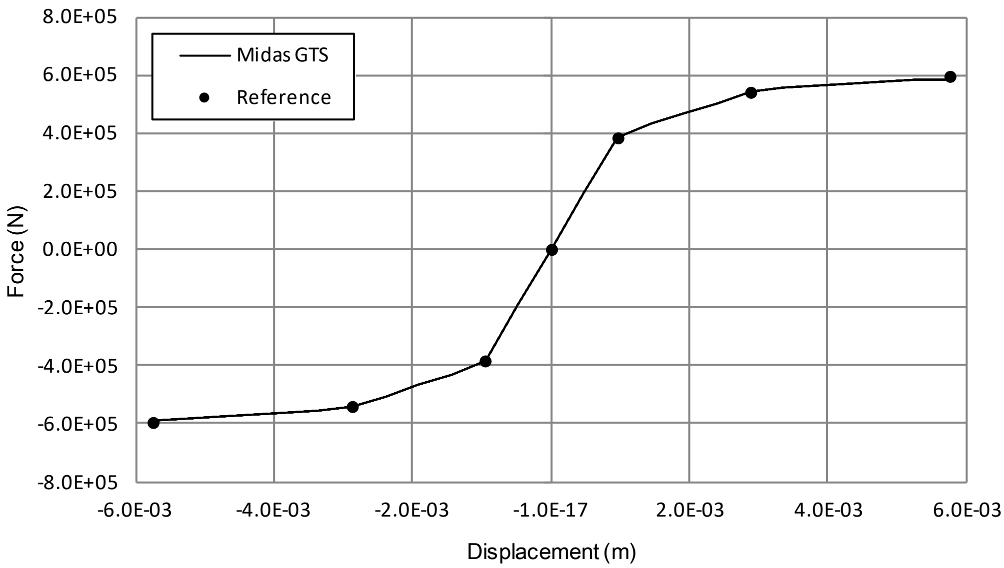


Table 7.1.1 Partial results of force-displacement relationship

Displacement	Force	
	Midas FEA	Reference
-5.76E-03	*-5.96E+05*	*-5.96E+05*
-2.88E-03	*-5.41E+05*	*-5.41E+05*
-9.60E-04	*-3.84E+05*	*-3.84E+05*
0.00E+00	*0.00E+00*	*0.00E+00*
9.60E-04	*3.84E+05*	*3.84E+05*
2.88E-03	*5.41E+05*	*5.41E+05*
5.76E-03	*5.96E+05*	*5.96E+05*

7.2 Kinematic hardening Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresis02.fea

Figure 7.2.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of kinematic hardening model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

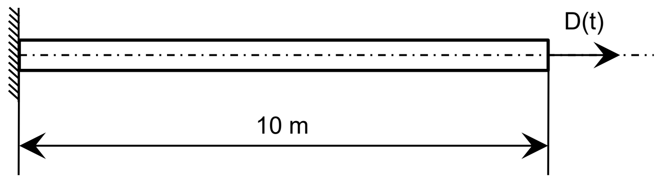


Figure 7.2.1
bar element model

Material data	Initial stiffness	$K = 4.0 \times 10^9\ N$
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.2.2
Skeleton curve data	1st Yield point	Force : $4.0 \times 10^5\ N$ Stiffness reduction : 0.1875
	2nd Yield point	Force : $5.5 \times 10^5\ N$ Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.2.2
Prescribed displacement at right end node

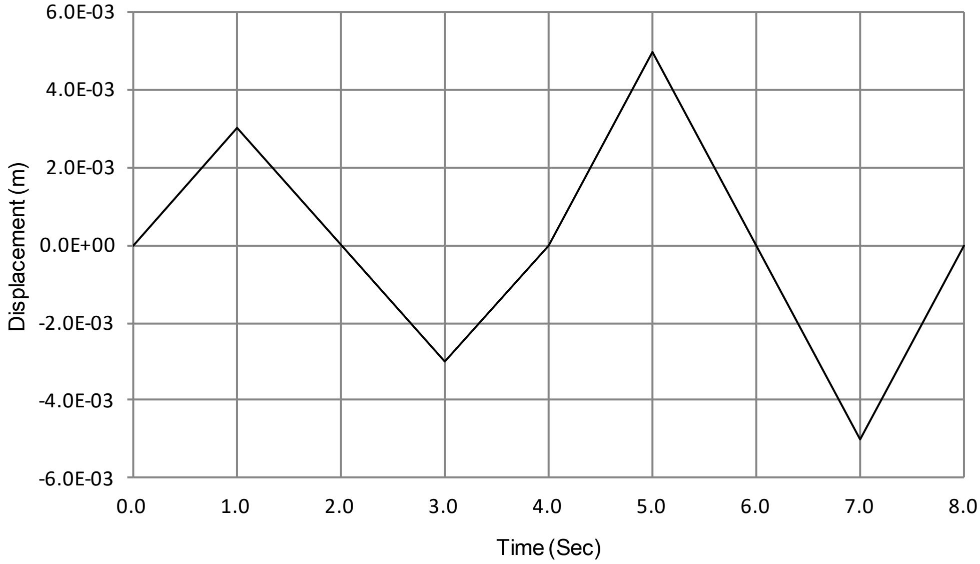


Figure 7.2.3
Force-displacement curve

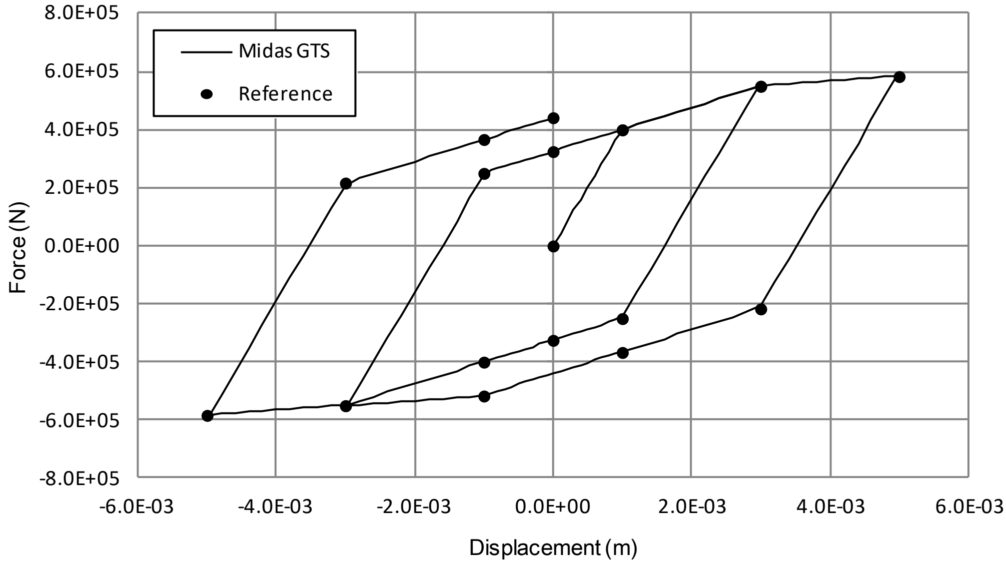


Table 7.2.1 Partial results of force-displacement relationship

	Force	
Displacement	Midas FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
1.00E-03	*-2.50E+05*	*-2.50E+05*
0.00E+00	*-3.25E+05*	*-3.25E+05*
-1.00E-03	*-4.00E+05*	*-4.00E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-1.00E-03	*2.50E+05*	*2.50E+05*
0.00E+00	*3.25E+05*	*3.25E+05*
1.00E-03	*4.00E+05*	*4.00E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
5.00E-03	*5.83E+05*	*5.83E+05*
3.00E-03	*-2.17E+05*	*-2.17E+05*
1.00E-03	*-3.67E+05*	*-3.67E+05*

7.3 Peak-Oriented model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys03.fea

Figure 7.3.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of peak-oriented model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

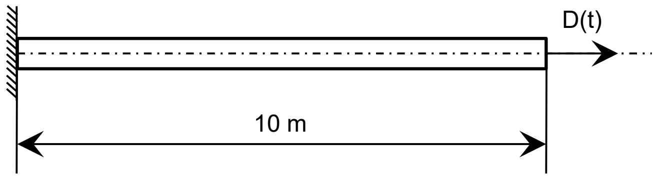


Figure 7.3.1
bar element model

Material data	Initial stiffness	$K = 4.0 \times 10^9$ N
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.3.2
Force-Stiffness reduction factor	1st Yield point	Force : 4.0×10^5 N Stiffness reduction : 0.1875
	2nd Yield point	Force : 5.5×10^5 N Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.3.2
Prescribed displacement at right end node

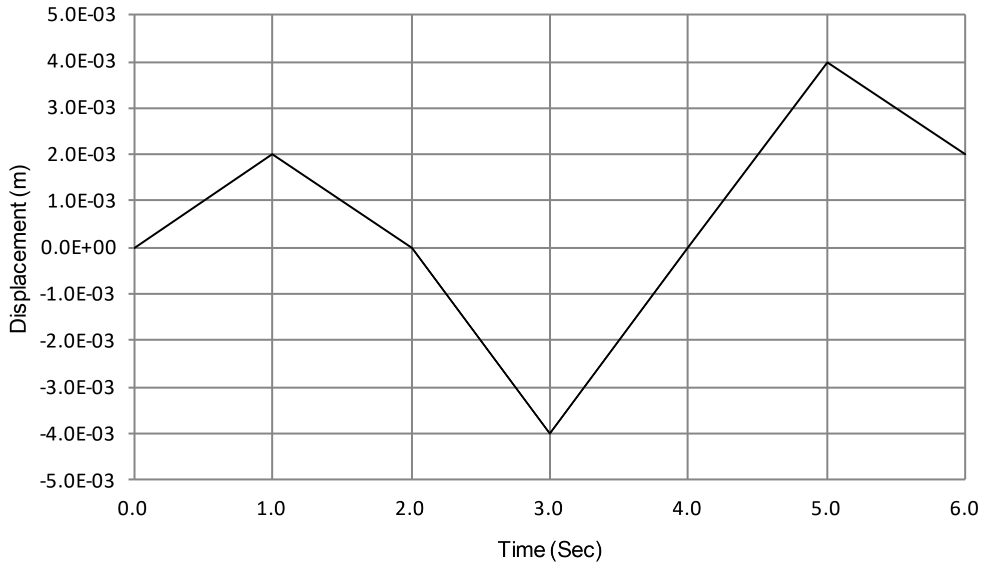


Figure 7.3.3
Force-displacement curve

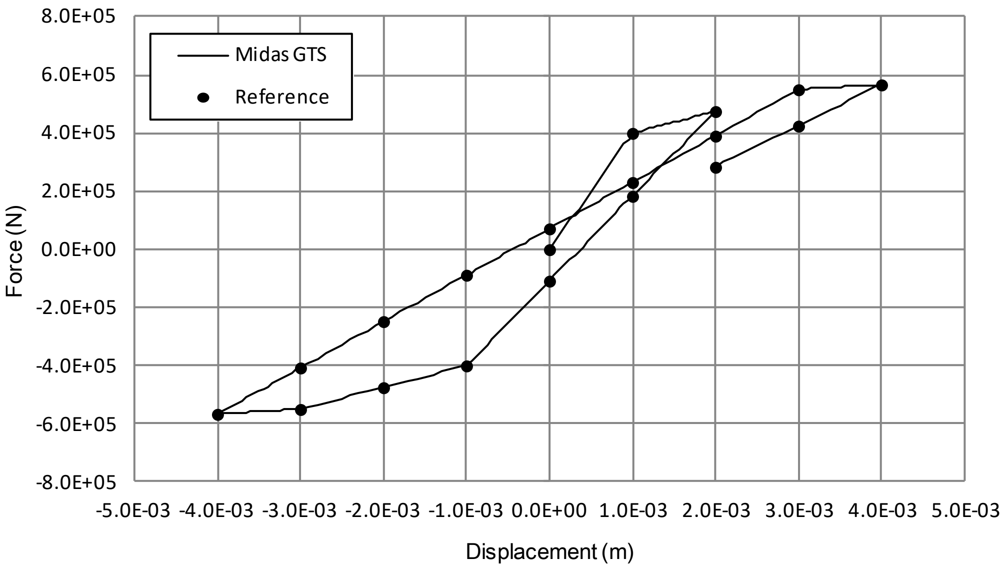


Table 7.3.1 Partial results of force-displacement relationship

	Force	
Displacement	Midas FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
2.00E-03	*4.75E+05*	*4.75E+05*
1.00E-03	*1.83E+05*	*1.83E+05*
0.00E+00	*-1.08E+05*	*-1.08E+05*
-1.00E-03	*-4.00E+05*	*-4.00E+05*
-2.00E-03	*-4.75E+05*	*-4.75E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-4.00E-03	*-5.67E+05*	*-5.67E+05*
-3.00E-03	*-4.07E+05*	*-4.07E+05*
-2.00E-03	*-2.48E+05*	*-2.48E+05*
-1.00E-03	*-8.81E+04*	*-8.81E+04*
0.00E+00	*7.14E+04*	*7.14E+04*
1.00E-03	*2.31E+05*	*2.31E+05*

7.4 Origin-Oriented model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys04.fea

Figure 7.4.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of origin-oriented model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

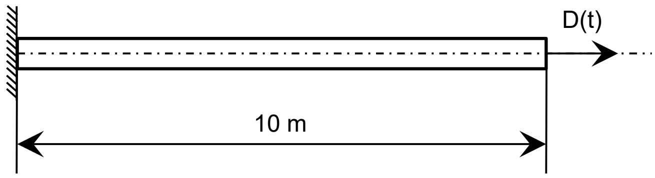


Figure 7.4.1
bar element model

Material data	Initial stiffness	$K = 4.0 \times 10^9\ N$
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.4.2
Force-Stiffness reduction factor	1st Yield point	Force : $4.0 \times 10^5\ N$ Stiffness reduction : 0.1875
	2nd Yield point	Force : $5.5 \times 10^5\ N$ Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

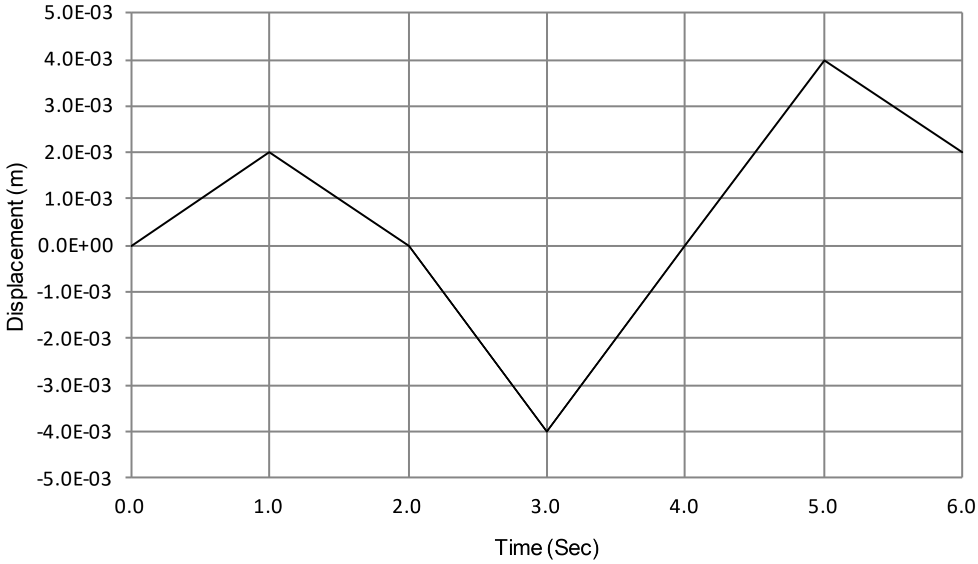


Figure 7.4.2
Prescribed displacement at right end node

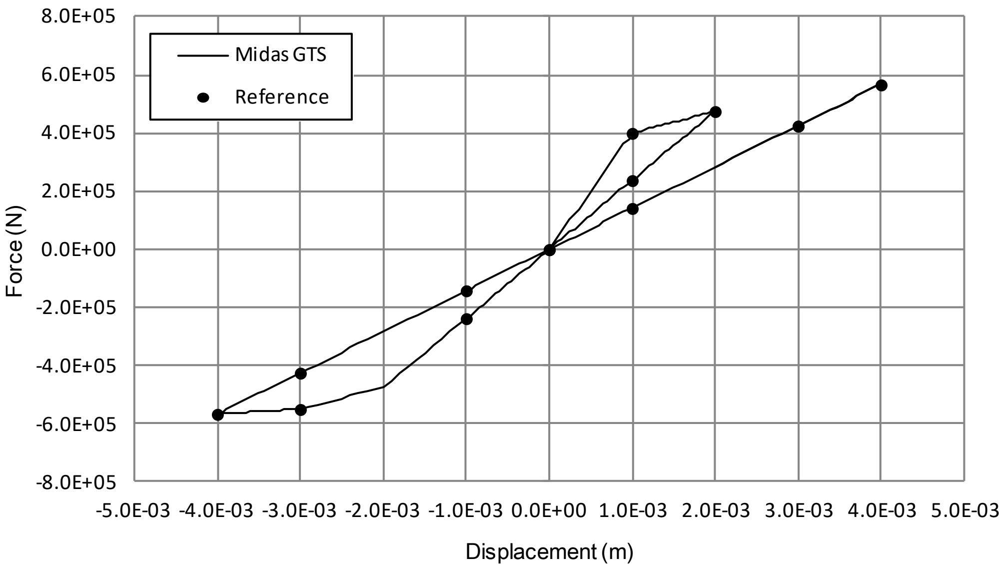


Figure 7.4.3
Force-displacement curve

Table 7.4.1 Partial results of force-displacement relationship

	Force	
Displacement	Midas FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
2.00E-03	*4.75E+05*	*4.75E+05*
1.00E-03	*2.38E+05*	*2.38E+05*
0.00E+00	*1.65E-09*	*1.65E-09*
-1.00E-03	*-2.38E+05*	*-2.38E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-4.00E-03	*-5.67E+05*	*-5.67E+05*
-3.00E-03	*-4.25E+05*	*-4.25E+05*
-1.00E-03	*-1.42E+05*	*-1.42E+05*
0.00E+00	*-4.15E-09*	*-4.15E-09*
1.00E-03	*1.42E+05*	*1.42E+05*
3.00E-03	*4.25E+05*	*4.25E+05*
4.00E-03	*5.67E+05*	*5.67E+05*

7.5 Clough Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys05.fea

Figure 7.5.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of clough model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

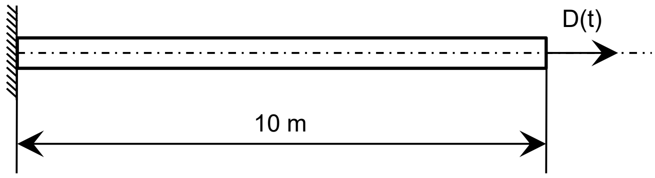


Figure 7.5.1
bar element model

Material data	Initial stiffness Unloading parameter	$K = 4.0 \times 10^9$ N β=0.2
Section property	Cross-sectional area	A =1 m^2
Loading	Prescribed displacement	Figure 7.5.2
Force-Stiffness reduction factor	1st Yield point	Force : 4.0×10^5 N Stiffness reduction : 0.1875
	2nd Yield point	Force : 5.5×10^5 N Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.5.2
Prescribed displacement at right end node

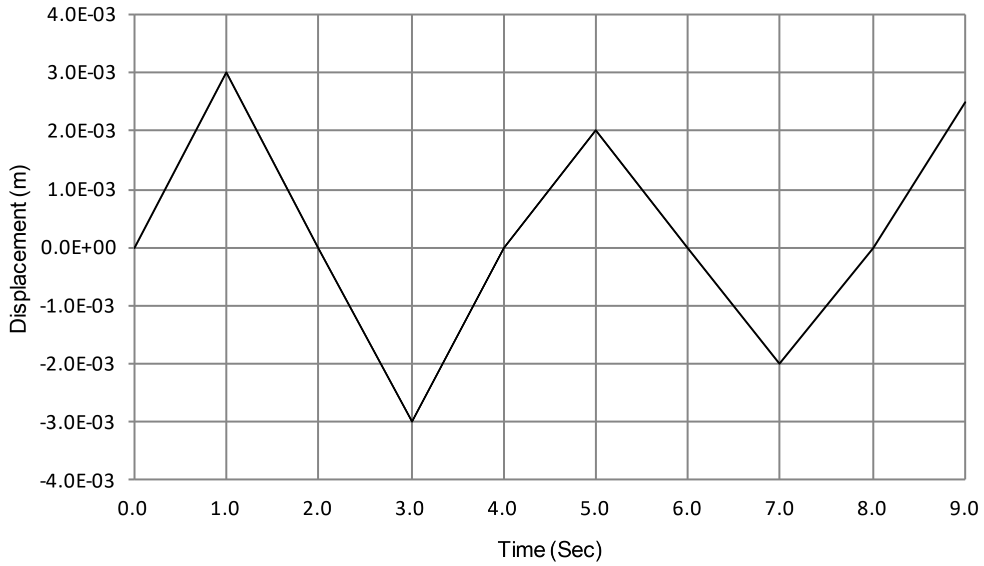


Figure 7.5.3
Force-displacement curve

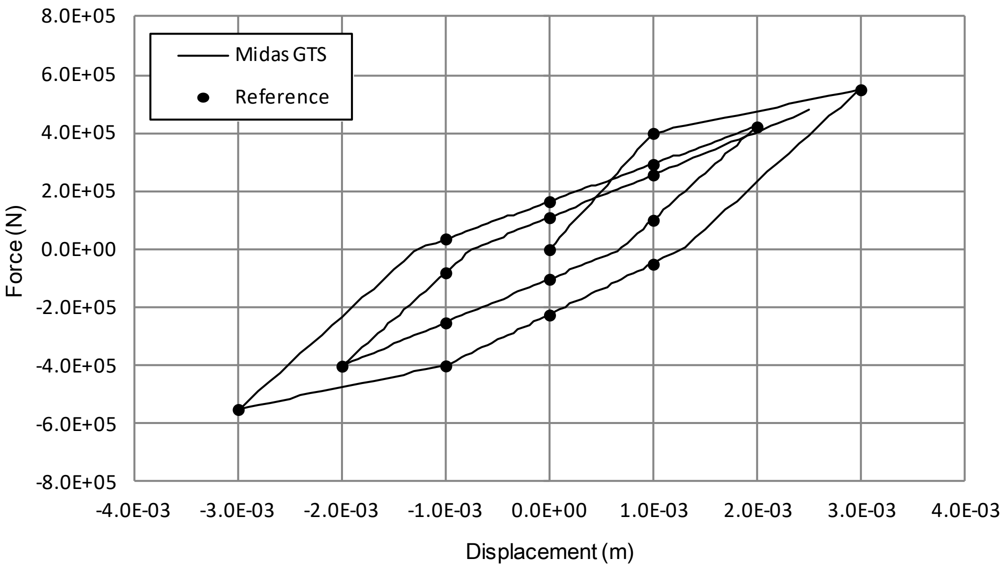


Table 7.5.1 Partial results of force-displacement

	Force	
Displacement	Midas FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
1.00E-03	*-5.02E+04*	*-5.02E+04*
0.00E+00	*-2.25E+05*	*-2.25E+05*
-1.00E-03	*-4.00E+05*	*-4.00E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-1.00E-03	*3.68E+04*	*3.68E+04*
0.00E+00	*1.65E+05*	*1.65E+05*
1.00E-03	*2.93E+05*	*2.93E+05*
2.00E-03	*4.22E+05*	*4.22E+05*
1.00E-03	*1.01E+05*	*1.01E+05*
0.00E+00	*-1.02E+05*	*-1.02E+05*
-1.00E-03	*-2.52E+05*	*-2.52E+05*

7.6 Degrading Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys06.fea

Figure 7.6.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of degrading model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

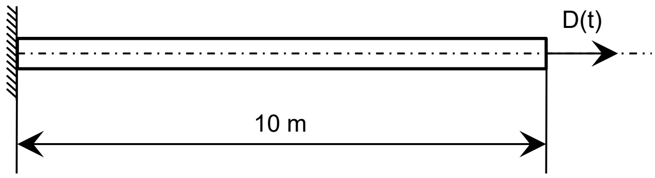


Figure 7.6.1
bar element model

Material data	Initial stiffness	$K = 4.0 \times 10^9\ N$
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.6.2
Force-Stiffness reduction factor	1st Yield point	Force : $4.0 \times 10^5\ N$ Stiffness reduction : 0.1875
	2nd Yield point	Force : $5.5 \times 10^5\ N$ Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.6.2
Prescribed displacement at right end node

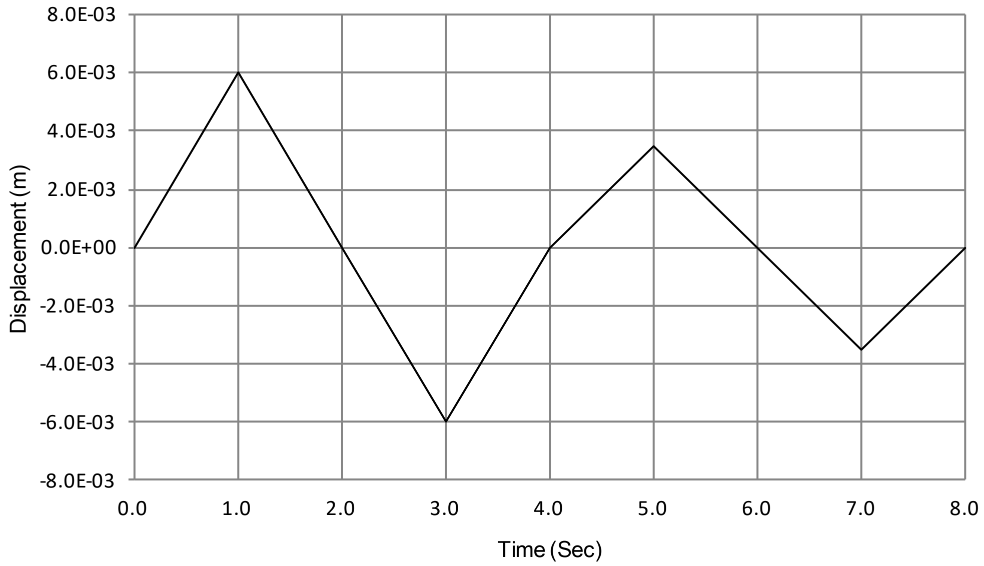


Figure 7.6.3
Force-displacement curve

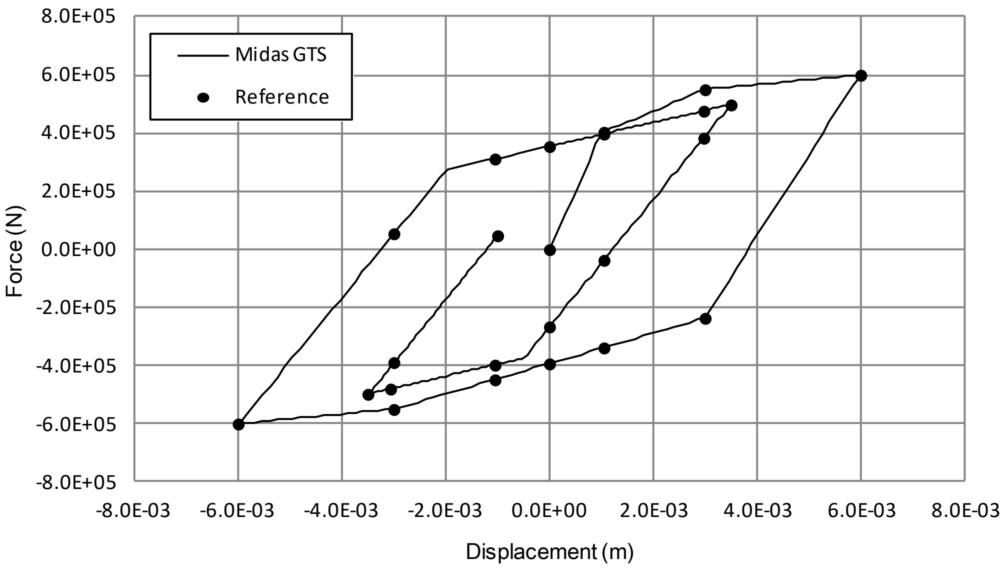


Table 7.6.1 Partial results of force-displacement

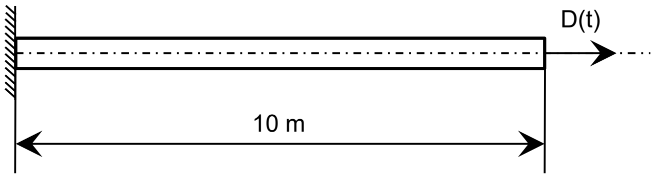
	Force	
Displacement	MIDAS FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.05E-03	*4.04E+05*	*4.04E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
6.00E-03	*6.00E+05*	*6.00E+05*
3.00E-03	*-2.36E+05*	*-2.36E+05*
1.05E-03	*-3.38E+05*	*-3.38E+05*
0.00E+00	*-3.93E+05*	*-3.93E+05*
-1.05E-03	*-4.48E+05*	*-4.48E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-6.00E-03	*-6.00E+05*	*-6.00E+05*
-3.00E-03	*5.45E+04*	*5.45E+04*
-1.05E-03	*3.12E+05*	*3.12E+05*
0.00E+00	*3.55E+05*	*3.55E+05*
1.05E-03	*3.98E+05*	*3.98E+05*

7.7 Takeda Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys07A.fea Hysteresys07B.fea

Figure 7.7.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of Takeda model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

Figure 7.7.1
bar element model



Material data	Initial stiffness Unloading parameter Inner loop parameter	$K = 4.0 \times 10^9$ N β=0.4 α=0.85
Section property	Cross-sectional area	A =1 m^2
Loading	Prescribed displacement	Figure 7.7.2 Figure 7.7.3
Force-Stiffness reduction factor	1st Yield point	Force : 4.0×10^5 N Stiffness reduction : 0.1875
	2nd Yield point	Force : 5.5×10^5 N Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.7.2
Prescribed
displacement at right
end node (case A)

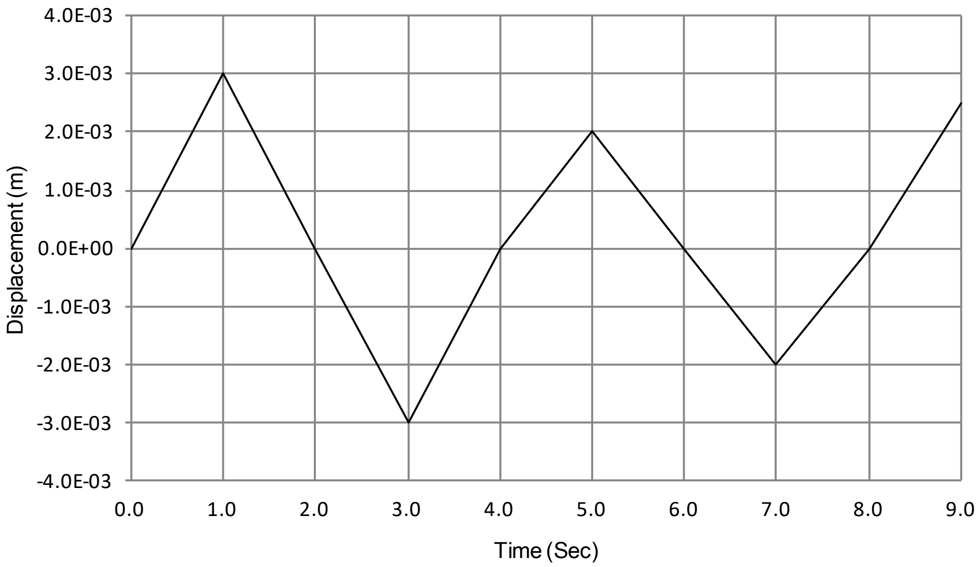


Figure 7.7.3
Prescribed
displacement at right
end node (case B)

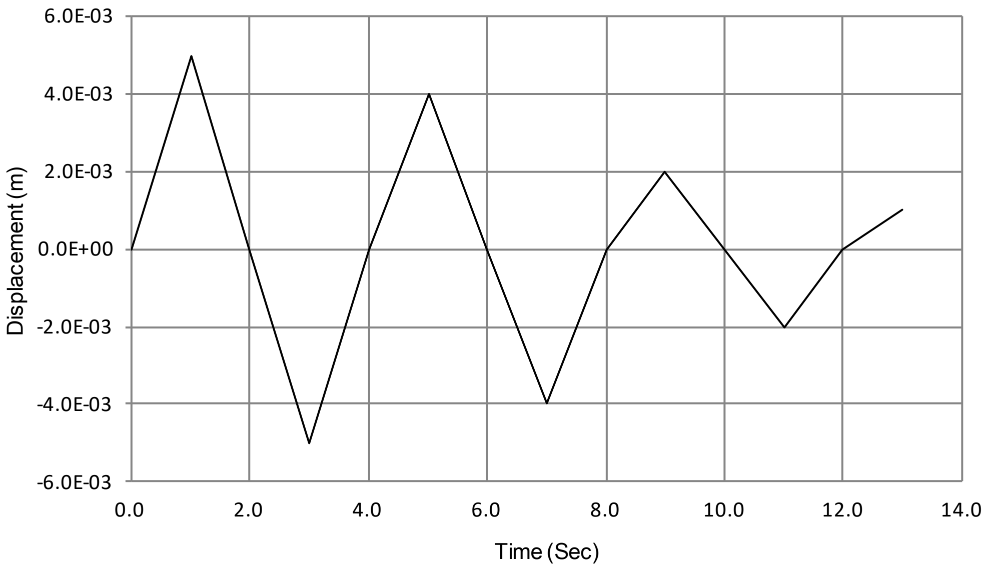


Figure 7.7.4
Force-displacement curve (case A)

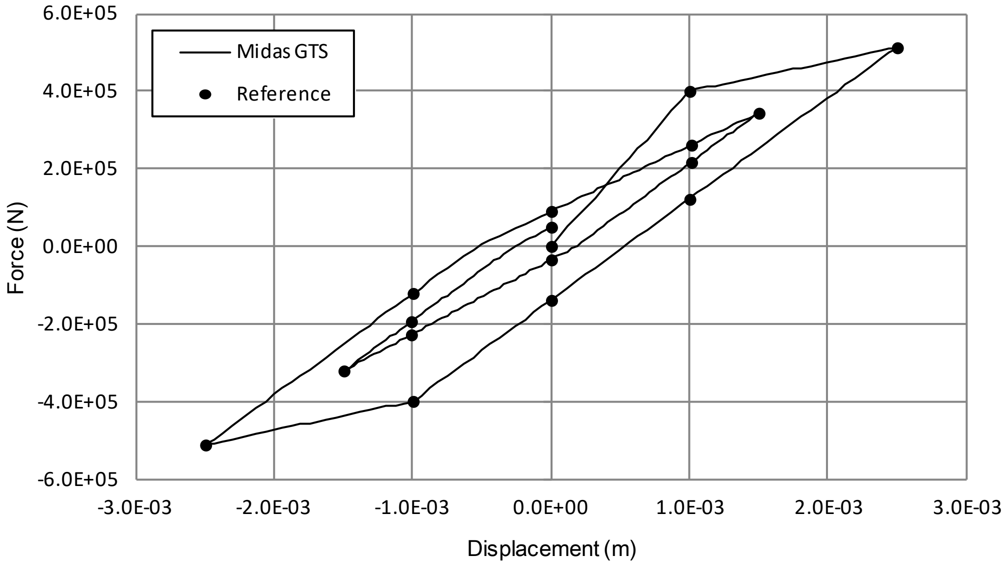


Figure 7.7.5
Force-displacement curve (case B)

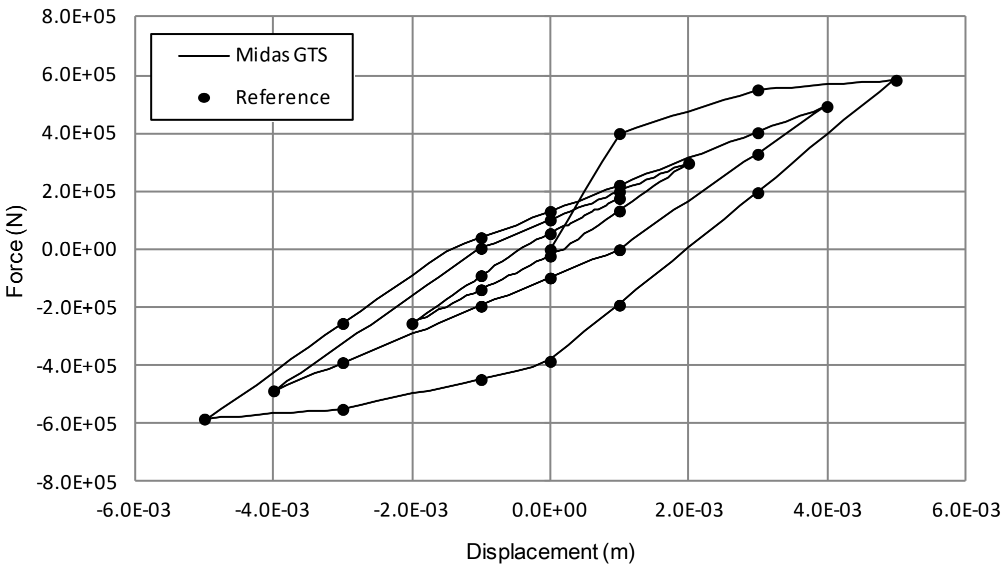


Table 7.7.1 Partial results of force-displacement (case A)

	Force	
Displacement	MIDAS FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
2.50E-03	*5.13E+05*	*5.13E+05*
1.00E-03	*1.21E+05*	*1.21E+05*
0.00E+00	*-1.39E+05*	*-1.39E+05*
-1.00E-03	*-4.00E+05*	*-4.00E+05*
-2.50E-03	*-5.13E+05*	*-5.13E+05*
-1.00E-03	*-1.21E+05*	*-1.21E+05*
0.00E+00	*9.02E+04*	*9.02E+04*
1.01E-03	*2.61E+05*	*2.61E+05*
1.50E-03	*3.44E+05*	*3.44E+05*
1.01E-03	*2.16E+05*	*2.16E+05*
0.00E+00	*-3.48E+04*	*-3.48E+04*
-1.01E-03	*-2.28E+05*	*-2.28E+05*
-1.50E-03	*-3.21E+05*	*-3.21E+05*

Table 7.7.2 Partial results of force-displacement (case B)

	Force	
Displacement	MIDAS FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
5.00E-03	*5.83E+05*	*5.83E+05*
3.00E-03	*1.96E+05*	*1.96E+05*
1.00E-03	*-1.91E+05*	*-1.91E+05*
0.00E+00	*-3.85E+05*	*-3.85E+05*
-1.00E-03	*-4.47E+05*	*-4.47E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-5.00E-03	*-5.83E+05*	*-5.83E+05*
-3.00E-03	*-2.54E+05*	*-2.54E+05*
-1.00E-03	*4.11E+04*	*4.11E+04*
0.00E+00	*1.32E+05*	*1.32E+05*
1.00E-03	*2.22E+05*	*2.22E+05*
3.00E-03	*4.03E+05*	*4.03E+05*

7.8 Modified Takeda Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys08.fea

Figure 7.8.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of modified Takeda model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

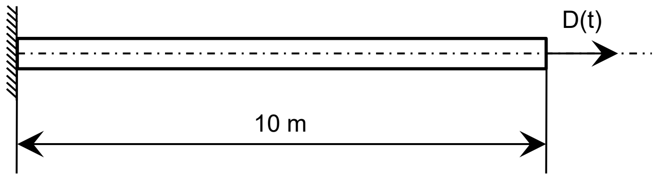


Figure 7.8.1
bar element model

Material data	Initial stiffness Unloading parameter Inner loop parameter	$K = 4.0 \times 10^9$ N β=0.4 α=0.85
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.8.2
Force-Stiffness reduction factor	1st Yield point	Force : 4.0×10^5 N Stiffness reduction : 0.1875
	2nd Yield point	Force : 5.5×10^5 N Stiffness reduction : 0.041667
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.8.2
Prescribed displacement at right end node

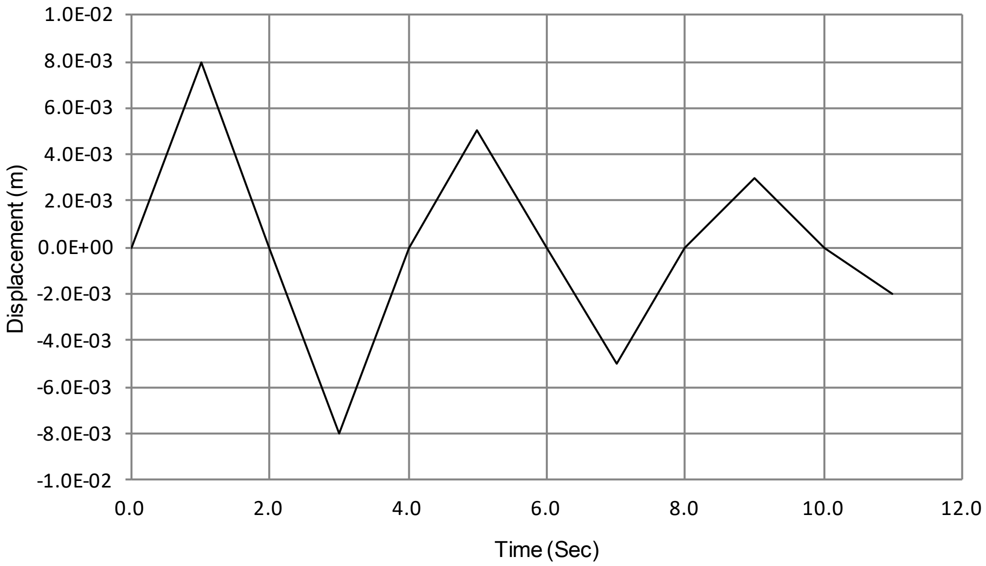


Figure 7.8.3
Force-displacement curve

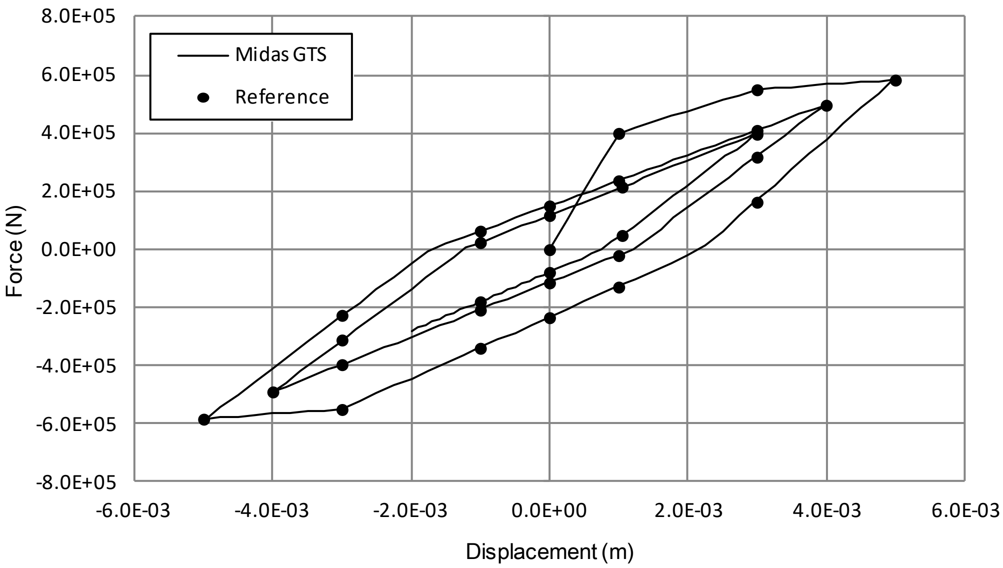


Table 7.8.1 Partial results of force-displacement

	Force	
Displacement	Midas FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
1.00E-03	*4.00E+05*	*4.00E+05*
3.00E-03	*5.50E+05*	*5.50E+05*
5.00E-03	*5.83E+05*	*5.83E+05*
3.00E-03	*1.63E+05*	*1.63E+05*
1.00E-03	*-1.29E+05*	*-1.29E+05*
0.00E+00	*-2.34E+05*	*-2.34E+05*
-1.00E-03	*-3.39E+05*	*-3.39E+05*
-3.00E-03	*-5.50E+05*	*-5.50E+05*
-5.00E-03	*-5.83E+05*	*-5.83E+05*
-3.00E-03	*-2.26E+05*	*-2.26E+05*
-1.00E-03	*6.36E+04*	*6.36E+04*
0.00E+00	*1.50E+05*	*1.50E+05*
1.00E-03	*2.37E+05*	*2.37E+05*
3.00E-03	*4.10E+05*	*4.10E+05*

7.9 Ramberg Osgood Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys09.fea

Figure 7.9.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of Ramberg Osgood model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

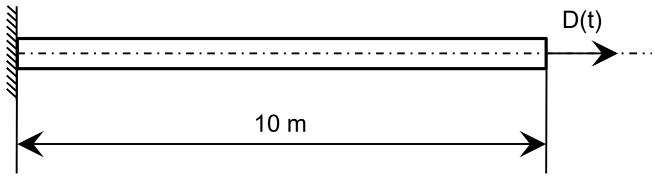


Figure 7.9.1
bar element model

Material data	Elastic modulus Critical displacement Maximum damping	$E = 210.0 \times 10^9\ N/m^2$ $\delta_r\ = 1.90476 \times 10^{-8}\ m$ h_{max}=0.12732
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.9.2
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.8.2
Prescribed displacement at right end node

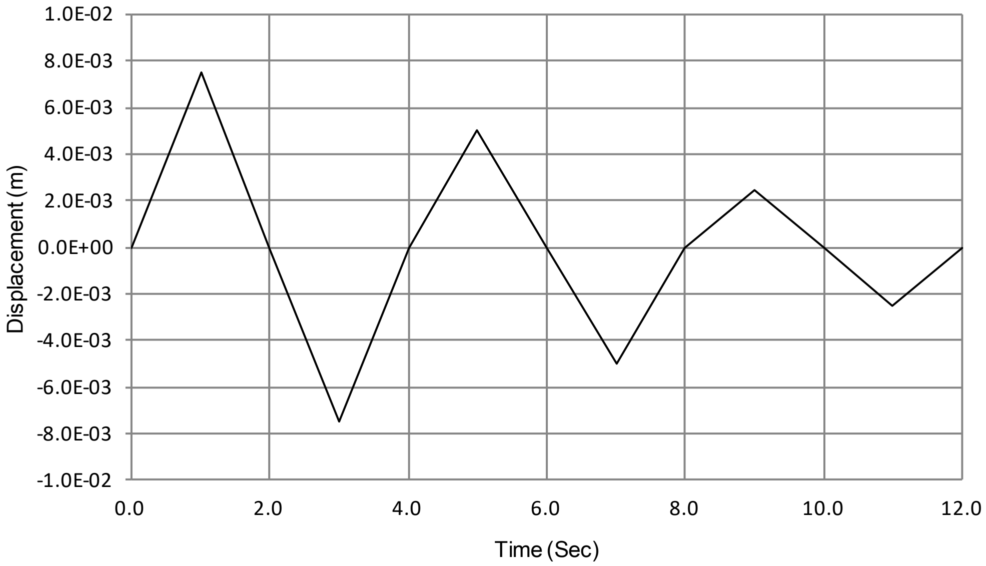


Figure 7.8.3
Force-displacement curve

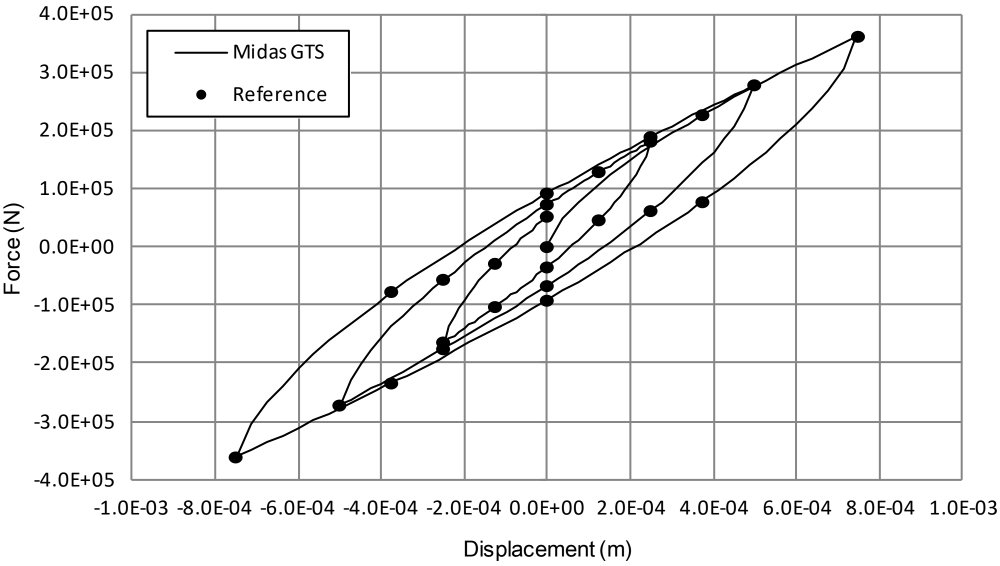


Table 7.9.1 Partial results of force-displacement

	Force	
Displacement	MIDAS FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
3.75E-04	*2.27E+05*	*2.27E+05*
7.50E-04	*3.62E+05*	*3.62E+05*
3.75E-04	*7.72E+04*	*7.72E+04*
0.00E+00	*-9.22E+04*	*-9.22E+04*
-3.75E-04	*-2.35E+05*	*-2.35E+05*
-7.50E-04	*-3.62E+05*	*-3.62E+05*
-3.75E-04	*-7.72E+04*	*-7.72E+04*
0.00E+00	*9.22E+04*	*9.22E+04*
2.50E-04	*1.89E+05*	*1.89E+05*
5.00E-04	*2.78E+05*	*2.78E+05*
2.50E-04	*6.20E+04*	*6.20E+04*
0.00E+00	*-6.71E+04*	*-6.71E+04*
-2.50E-04	*-1.76E+05*	*-1.76E+05*
-5.00E-04	*-2.73E+05*	*-2.73E+05*

7.10 Hardin Drnevich Model

REFERENCE	Analytical solution
ELEMENT	Bar element
MODEL FILENAME	Hysteresys10.fea

Figure 7.10.1 shows a various Bar element model subjected to axial displacement dependent on time. The fixed boundary condition is assigned to the left end. In case of Hardin Drnevich model, hinge is considered in order to evaluate relationship between force and stiffness reduction factor at the hinge location.

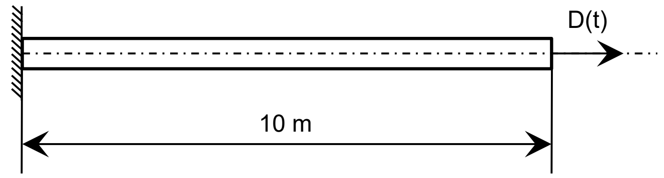


Figure 7.10.1
bar element model

Material data	Elastic modulus Critical displacement	$E = 210.0 \times 10^9\ N/m^2$ $\delta_r = 9.5238 \times 10^{-4}\ m$
Section property	Cross-sectional area	$A = 1\ m^2$
Loading	Prescribed displacement	Figure 7.10.2
Hinge location	Distributed plasticity	Gauss-Lobatto integration point

Figure 7.10.2
Prescribed displacement at right end node

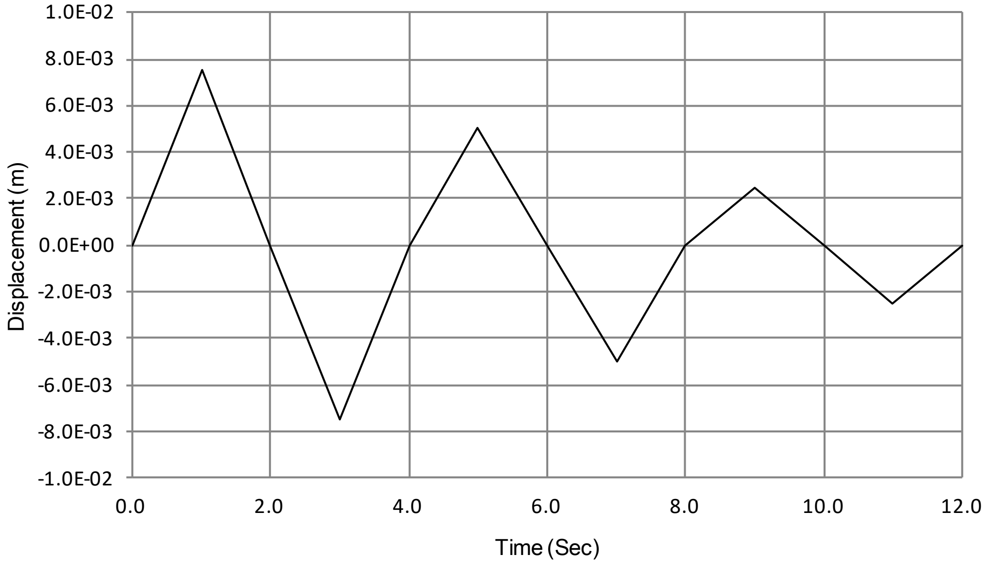


Figure 7.10.3
Force-displacement curve

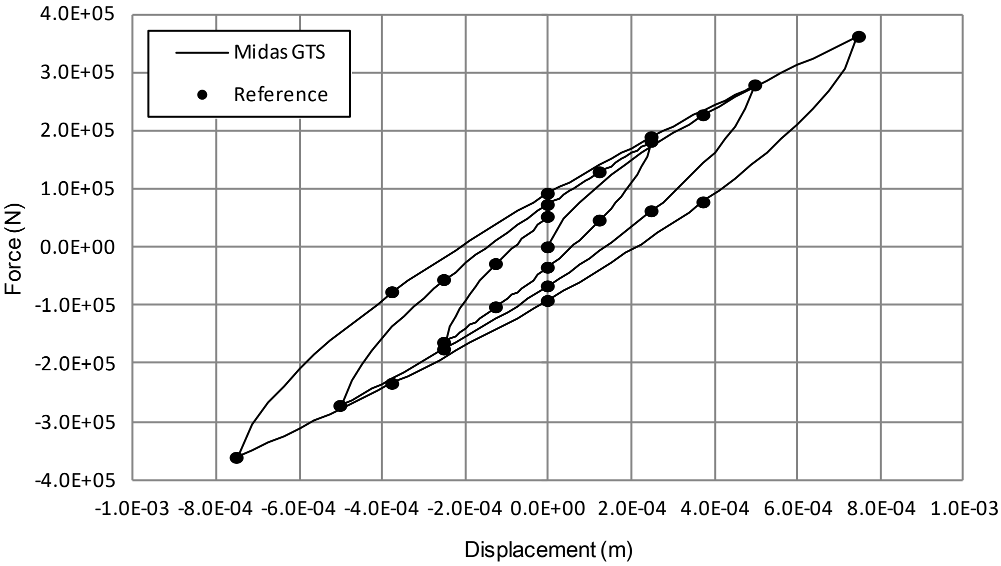


Table 7.10.1 Partial results of force-displacement

	Force	
Displacement	MIDAS FEA	Reference
0.00E+00	*0.00E+00*	*0.00E+00*
3.75E-04	*5.65E+06*	*5.65E+06*
7.50E-04	*8.81E+06*	*8.81E+06*
3.75E-04	*2.23E+06*	*2.23E+06*
0.00E+00	*-2.49E+06*	*-2.49E+06*
-3.75E-04	*-6.04E+06*	*-6.04E+06*
-7.50E-04	*-8.81E+06*	*-8.81E+06*
-3.75E-04	*-2.23E+06*	*-2.23E+06*
0.00E+00	*2.49E+06*	*2.49E+06*
2.50E-04	*4.96E+06*	*4.96E+06*
5.00E-04	*7.04E+06*	*7.04E+06*
2.50E-04	*2.40E+06*	*2.40E+06*
0.00E+00	*-1.28E+06*	*-1.28E+06*
-2.50E-04	*-4.26E+06*	*-4.26E+06*
-5.00E-04	*-6.73E+06*	*-6.73E+06*