10.1 Contact Patch test

REFERENCE	NAFEMS[1]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis01.fea

Figure 10.1.1 shows a frictionless two-body model for a contact patch test. The bottom side of the foundation is constrained in the vertical direction, with the center of this side constrained in all directions. Uniform pressure of 0.04 N/mm^2 is applied on the top surface of the body 1. Nonlinear analysis is carried out to obtain contact pressure on the contact interfaces with various regular and irregular meshes. The effect of geometric nonlinearity is neglected.

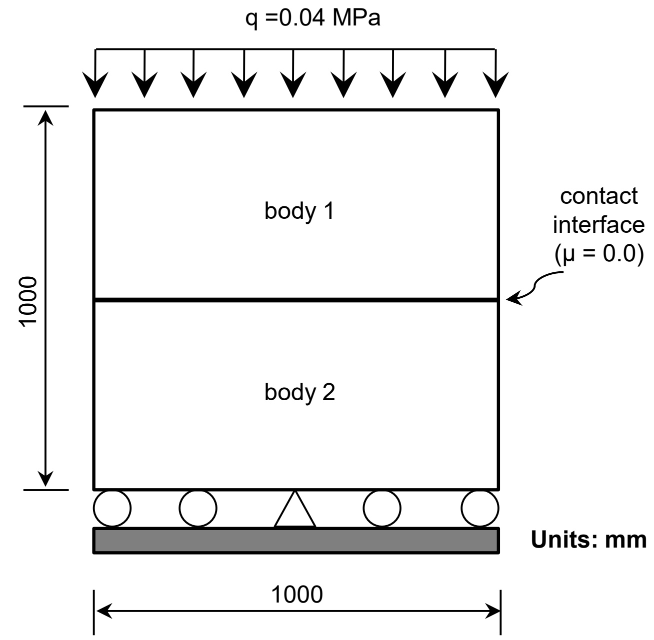


Figure 10.1.1
Two-body contact model

Material data	Young's modulus Poisson's ratio	$E_{punch} = E_{foundation} = 100$ MPa $\nu_{punch} = \nu_{foundation} = 0.3$

Figure 10.1.2
Various meshes from top left clockwise; regular matching, regular non-matching, irregular matching and irregular non-matching

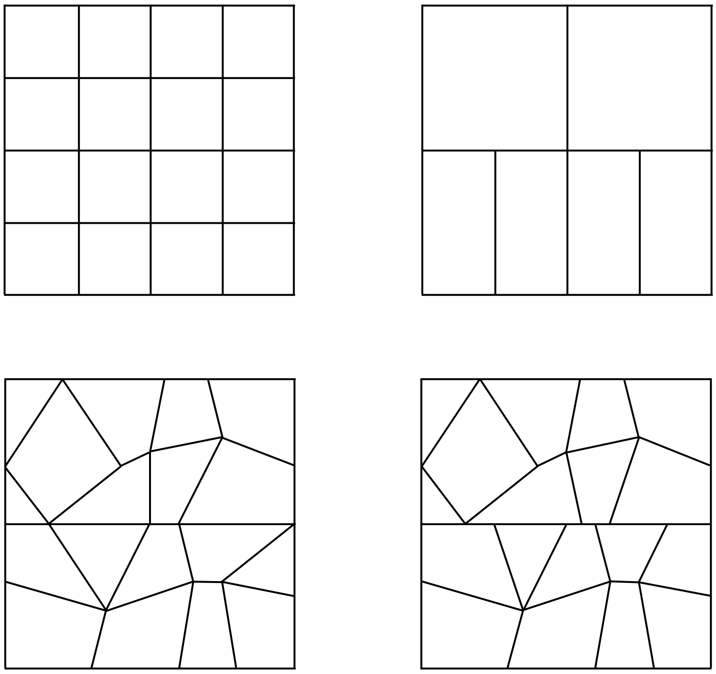
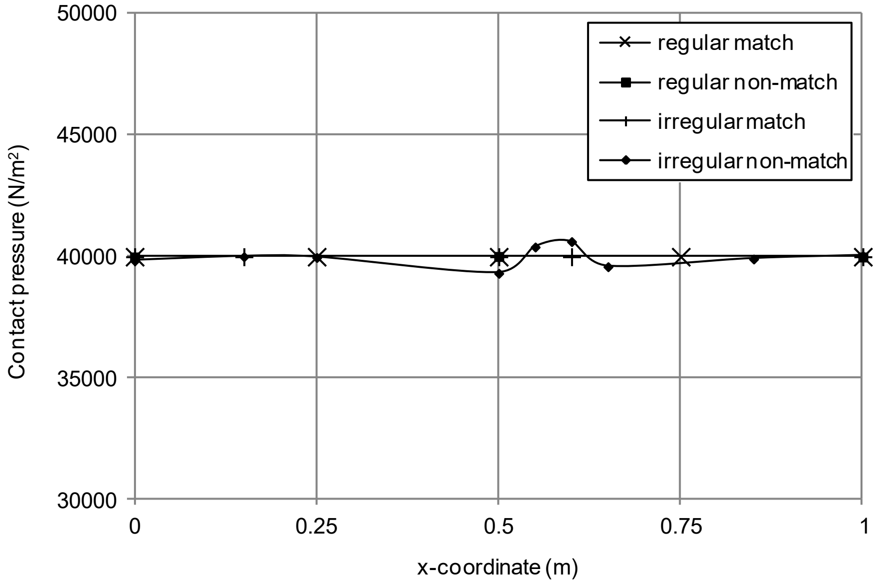


Figure 10.1.3
Contact pressure distribution obtained using various meshes

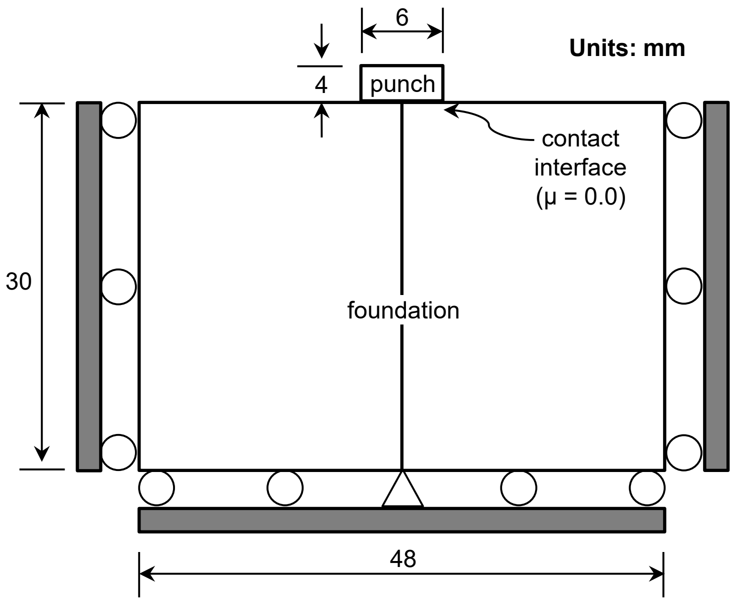


10.2 Rigid punch contact

REFERENCE	Gladwell [2]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis02.fea

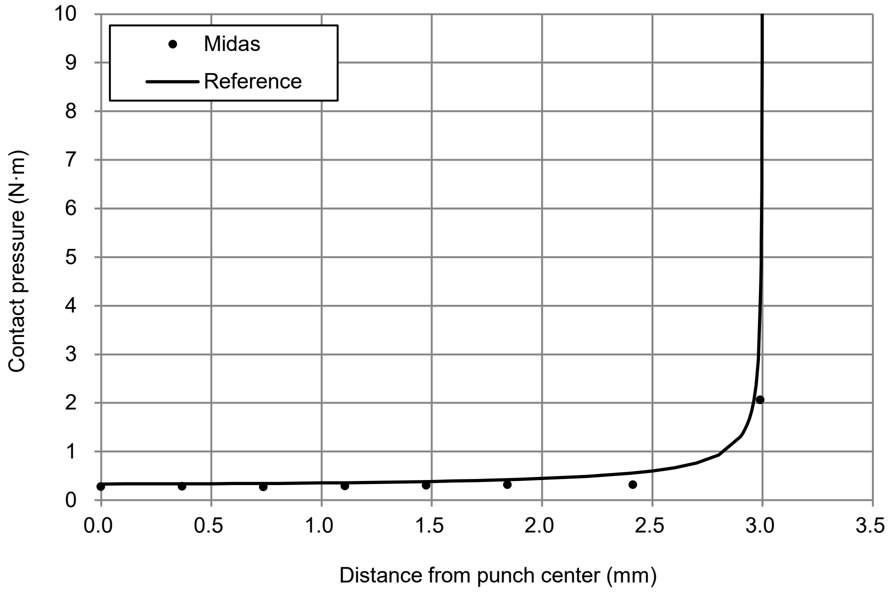
Figure 10.2.1 shows a rigid punch pressed into a frictionless elastic foundation. The bottom surface of the foundation is constrained in the vertical direction with the center of the surface constrained in all directions. The vertical sides of the foundation are constrained in the horizontal direction. A prescribed vertical displacement of 0.05 mm is applied to the punch. Contact pressure is determined on the contact interface and compared with the theoretical solution. The results are obtained without considering geometric nonlinear effects.

Figure 10.2.1
Rigid punch contact model



Material data	Young's modulus Poisson's ratio	$E_{punch} = E_{foundation} = 100\ MPa$ $\nu_{punch} = \nu_{foundation} = 0.3$

Figure 10.2.2
Contact pressure
distribution (3D)

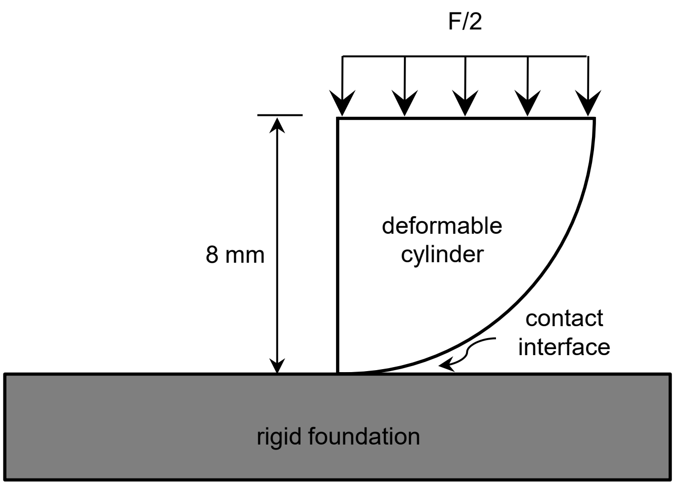


10.3 Hertzian contact

REFERENCE	NAFEMS[1], Gladwell[2]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis03.fea

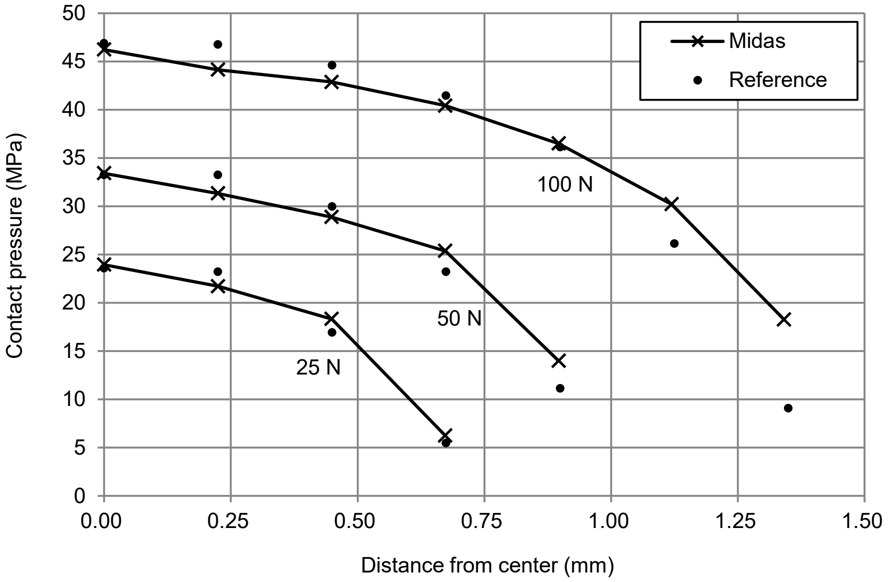
Figure 10.3.1 shows an elastic cylinder pressed onto a rigid foundation. The rigid foundation is constrained in all directions. The cylinder is constrained in the horizontal direction along the inner edge to simulate the behavior of a half cylinder. The results are obtained for three load values; F = 25, 50, and 100 N. Geometric nonlinear analyses are carried out to determine the contact pressure along the contact interface, and the results are compared with the theoretical solution.

Figure 10.3.1
Pressed elastic cylinder model



Material data	Young's modulus Poisson's ratio	E_{cyl} = 500 MPa ν = 0.3

Figure 10.3.2
Contact pressure distribution

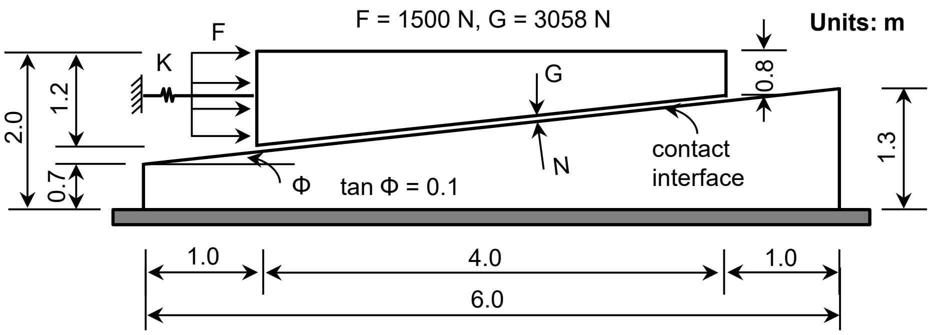


10.4 Sliding wedge with linear springs

REFERENCE	NAFEMS[1]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis04.fea

Figure 10.4.1 shows a sliding wedge on an elastic foundation under the gravity load. The bottom side of the foundation is constrained in all directions. The left side of the wedge is connected to 9 linear 2-node springs in the horizontal direction. Horizontal distances are determined for the four cases of frictional constants and spring stiffness as summarized in Table 10.4.1. Geometric nonlinear analyses are carried out to obtain the horizontal displacements, which are then compared with the reference values.

Figure 10.4.1
Sliding wedge model



Material data	Young's modulus	E = 206 GPa
	Poisson's ratio	ν = 0.3

Table 10.4.1 Horizontal displacements obtained for four different friction coefficients and spring stiffness values

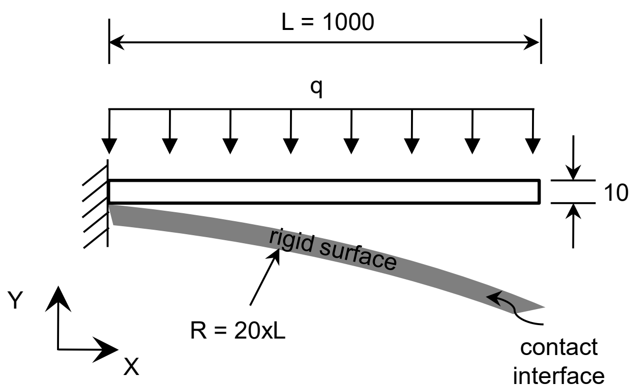
Friction coefficient	Force [N]	Spring stiffness [N/m]	Horizontal displacement [m]	
			Reference	Midas
0.0	1500	132.6	1.0	1.0
0.1	1500	98.0	1.0	1.0
0.2	1500	62.6	1.0	1.0
0.3	1500	26.5	1.0	1.0

10.5 Cantilever beam and a rigid curvilinear surface

REFERENCE	NAFEMS[1], Ayari et al.[3]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis05.fea

Figure 10.5.1 shows a cantilever beam loaded by a uniform pressure load over a frictionless rigid curvilinear surface. The rigid surface is constrained in all directions at its reference node. The left side of the cantilever is constrained in all directions. The vertical deflection of the cantilever tip and the horizontal distance from the root to the point of separation are determined for three cases of pressure levels;   = 0.05, 0.075 and 0.1 N/mm2 by carrying out geometric nonlinear analyses.

Figure 10.5.1
Cantilever beam model



Material data	Young's modulus	*E = 206 GPa*	
	Poisson's ratio	$\nu = 0.3$	

Table 10.5.1 Vertical displacement at cantilever tip,   and horizontal distance from root to point of separation,

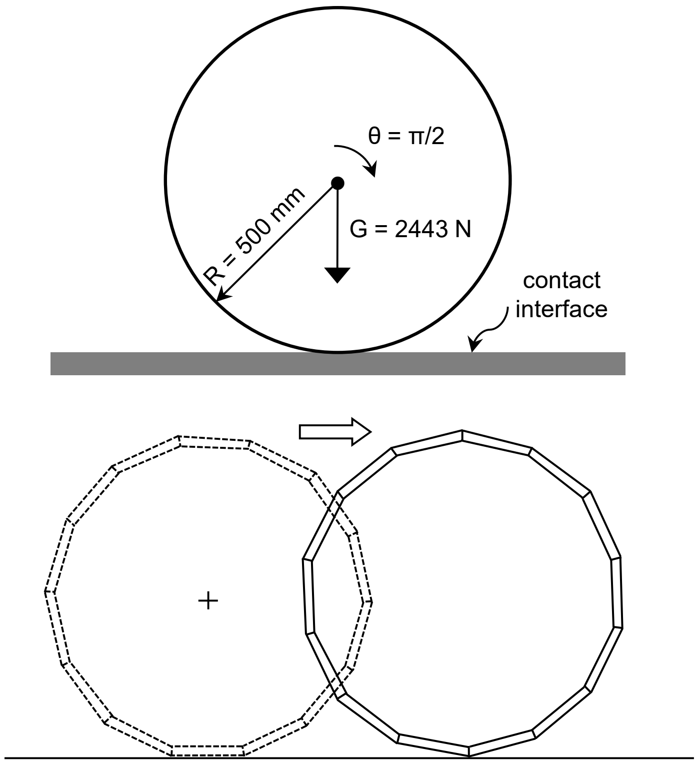
q [N/mm^2]		0.050	0.075	0.100
u_y [mm]	Reference	23.55	24.03	24.27
	Midas	20.60	22.05	22.81
x_S [mm]	Reference	410	518	583
	Midas	425	525	600

10.6 Sliding and rolling of a ring on a rigid surface

REFERENCE	NAFEMS[1]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis06.fea

A steel ring under the gravity load is allowed to slide and roll on a rigid surface, as depicted in Figure 10.6.1. The friction coefficient between the two bodies is 1.0. The rigid plate is constrained in all directions. Geometric nonlinear analysis is carried out to determine the horizontal displacement of the center point as the steel ring is rotated by 90 degrees. The horizontal displacement and the maximum vertical displacement of the center of the ring are compared to the theoretical solution.

Figure 10.6.1
Sliding and rolling of a ring



Material data	Density	*ρ = 7850 kg/m³*
	Young's modulus	*E = 206 GPa*
	Poisson's ratio	*ν = 0.3*

Table 10.6.1 Horizontal and maximum vertical displacements of center of the ring

	Reference	Midas
u_H [mm]	*785.398(circle)* *778.823(polygon)*	*778.818*
u_V [mm]	*12.536*	*12.536*

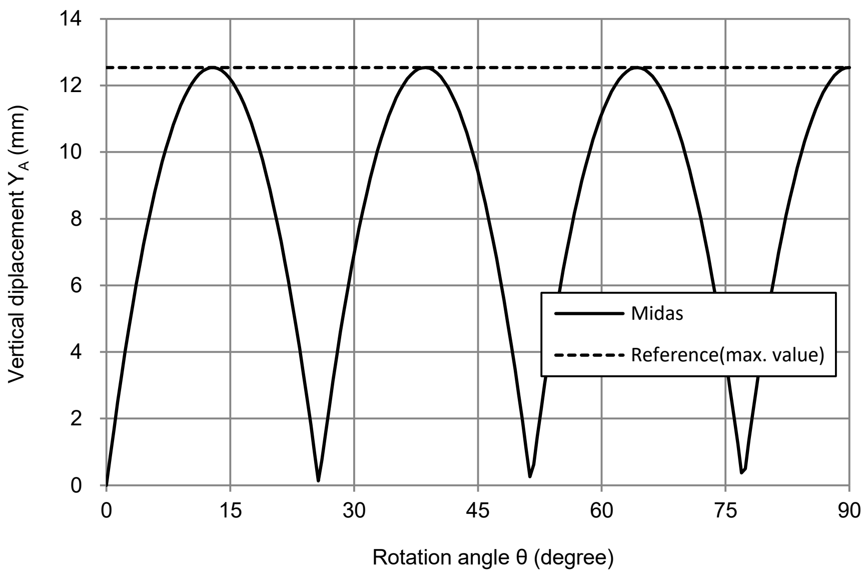
Horizontal displacement:

- *circle:* $u_H = 2\pi R / 4 = 785.398$
- *polygon:* $u_H = 7R\sin\left(0.5\pi / 7\right) = 778.823$

Maximum vertical displacement:

- $u_V = R\left[1-\cos\left(0.5\pi / 7\right)\right] = 12.536$

Figure 10.6.2
Contact pressure distribution

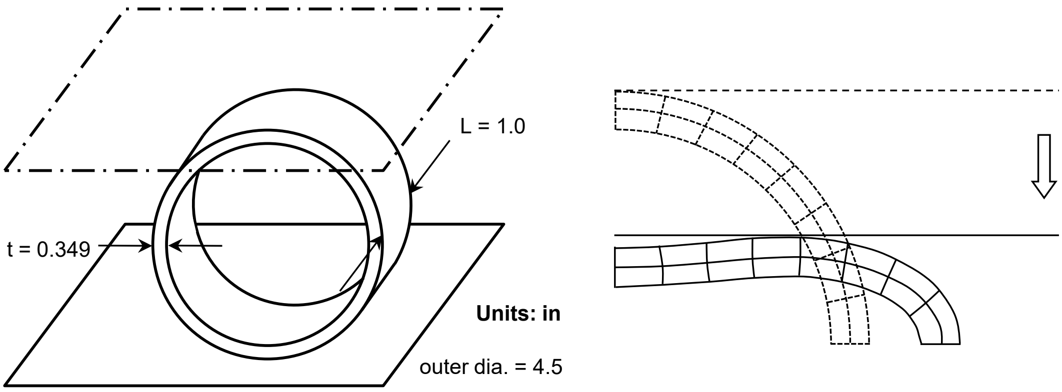


10.7 Crushing of a pipe

REFERENCE	Peech et al.[4]
ELEMENTS	Solid elements
MODEL FILENAME	ContactAnalysis07.fea

Figure 10.7.1 shows a pipe section to be crushed. The rigid plates are constrained in all directions. A quarter of the pipe section is modeled utilizing the symmetry, and the associated symmetric boundary conditions are applied. The pipe material is defined by the von Mises plastic with isotropic hardening. The plastic curve is given in Table 10.7.1. The reaction force per unit length is determined versus the relative anvil displacement.

Figure 10.7.1
Pipe section and its deformed shape



Material data	Young's modulus	$E = 206\ lbf/in^2$
	Poisson's ratio	$\nu = 0.3$

Table 10.7.1 Plastic Curve

Strain	Stress [*psi*]
0.00000	*35000*
0.00350	*40000*
0.00830	*43750*
0.01330	*46250*
0.01820	*50000*
0.02810	*52500*
0.03800	*55000*
1.00000	*215000*

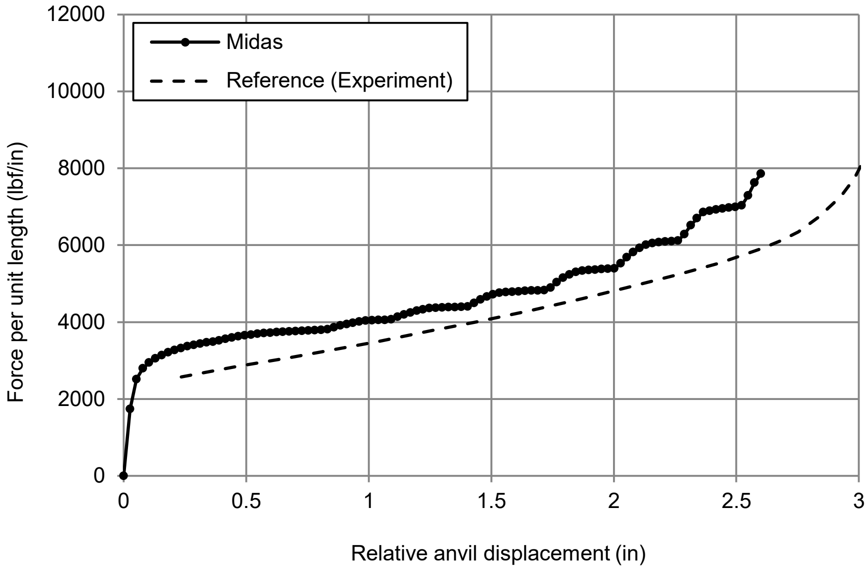


Figure 10.7.2
Vertical displacement
as the inner ring rotates

References

1 NAFEMS, The Standard NAFEMS Benchmarks, Rev. 3, NAFEMS, Glasgow, 1990

2 G.M.L. Gladwell, *Contact Problems in the Classical Theory of Elasticity*, Sijthoff and Noordhoff, Alphen aan den Rijn, 1980

3 M.L. Ayari and V.E. Saouma, “Static and Dynamic Contact/Impact Problems Using Fictitious Forces,” International Journal for Numerical Methods in Engineering, 32, pp. 623-643, 1991

4 J.M. Peech, R.E. Roener, S.D. Porofin, G.H. East, and N.A. Goldstein, "Local Crush Rigidity of Pipes and Elbows," Proc. 4th SMIRT Conf. paper F-3/8, North Holland, 1977