

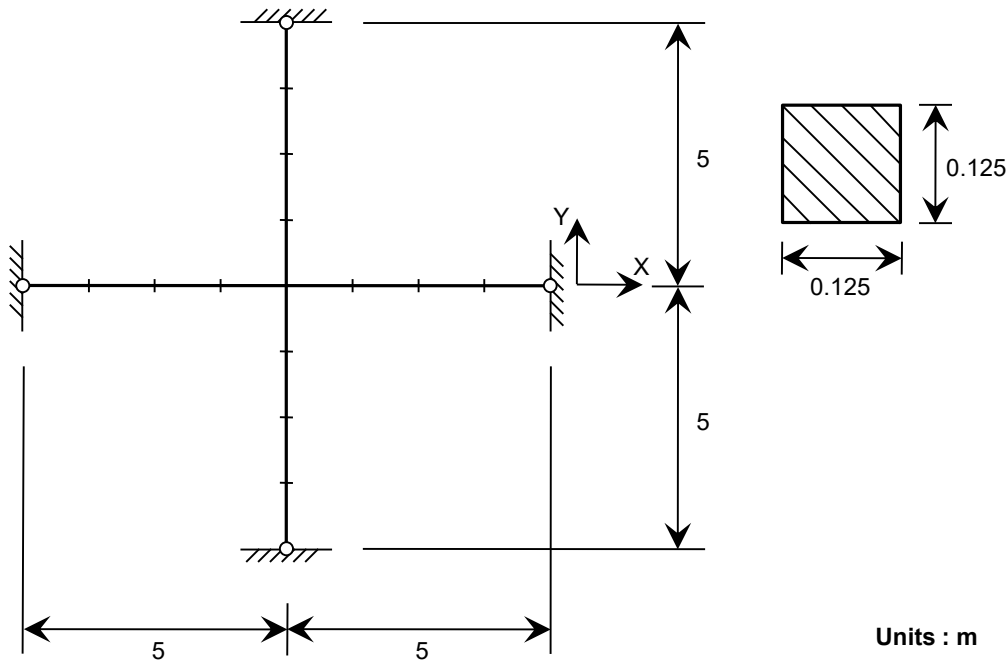


4.1 Pin-ended Cross

REFERENCE	NAFEMS ¹
ELEMENTS	Beam elements
MODEL FILENAME	Dynamics01.fea

Figure 4.1.1 shows a pin-ended cross model. The structure is modeled with 4 beam elements for each arm. Eigenvalue analysis is carried out. The natural frequencies of the lowest modes of which the vibration mode shapes are depicted in Figure 4.1.2 are compared with the reference values given in the NAFEMS benchmarks.

Figure 4.1.1
Pin-ended cross model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$
Section property	Square cross-section	$0.125 \text{ m} \times 0.125 \text{ m}$



Figure 4.1.2
Vibration mode shapes

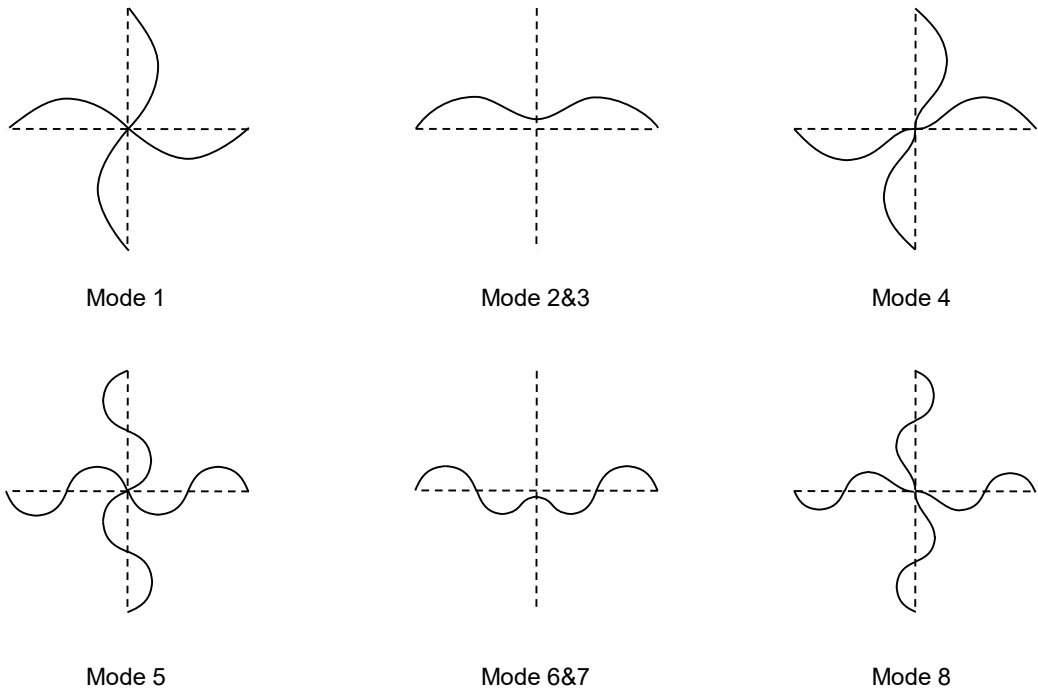


Table 4.1.1 Natural frequencies in Hz obtained using beam elements

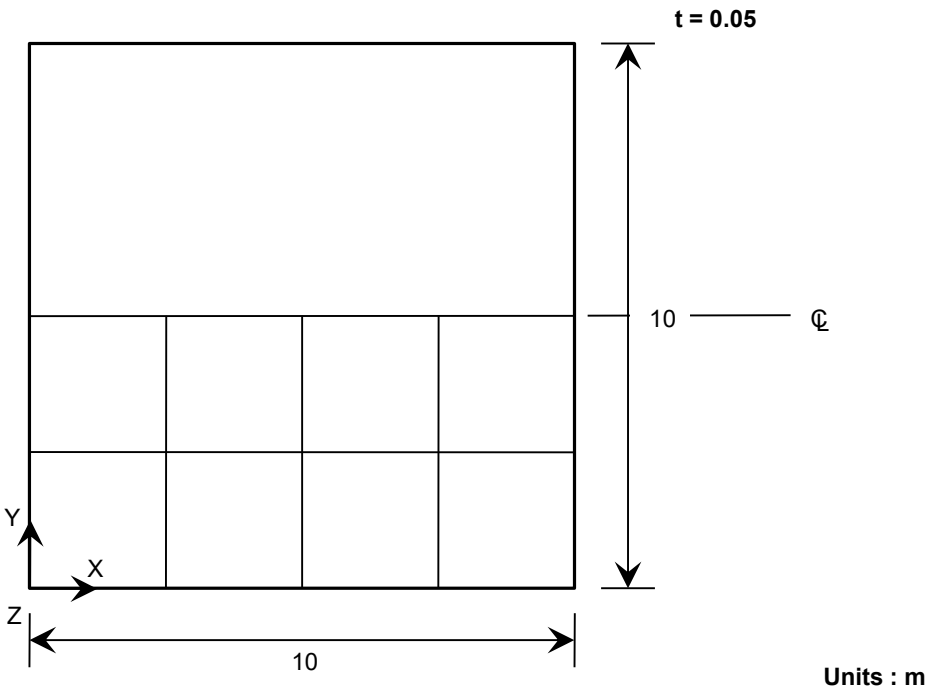
Mode Number		1	2, 3	4	5	6, 7	8
Reference		11.336	17.709	17.709	45.345	57.390	57.390
Element type	Number of elements						
Beam-2	4 per arm	11.338	17.689	17.717	45.483	57.371	57.690

4.2 Thin Square Cantilever Plate

REFERENCE	NAFEMS ¹
ELEMENTS	Shell elements, solid elements
MODEL FILENAME	Dynamics02(CaseA).fea Dynamics02(CaseB).fea

Figure 4.2.1 shows a thin square cantilever plate model. One half of the model is discretized using shell and solid elements. Depending on the applied boundary conditions, eigenvalue analyses of two sub-cases are conducted; Case A considers only the symmetric modes and Case B considers only the anti-symmetric vibration modes.

Figure 4.2.1
Thin square cantilever
plate model



- case A

$x = y = R_z = 0$ at all nodes,
 $z = R_y = R_x = 0$ along y-axis, $R_x = 0$ along $y = 5\text{ m}$
- case B

$x = y = R_z = 0$ at all nodes,
 $z = R_y = R_x = 0$ along y-axis

Material data	Young's modulus	$E = 200\text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532\text{ kN/m}^3$
Section property	Thickness	$t = 0.05\text{ m}$



Figure 4.2.2
Vibration mode shapes
(Case A)

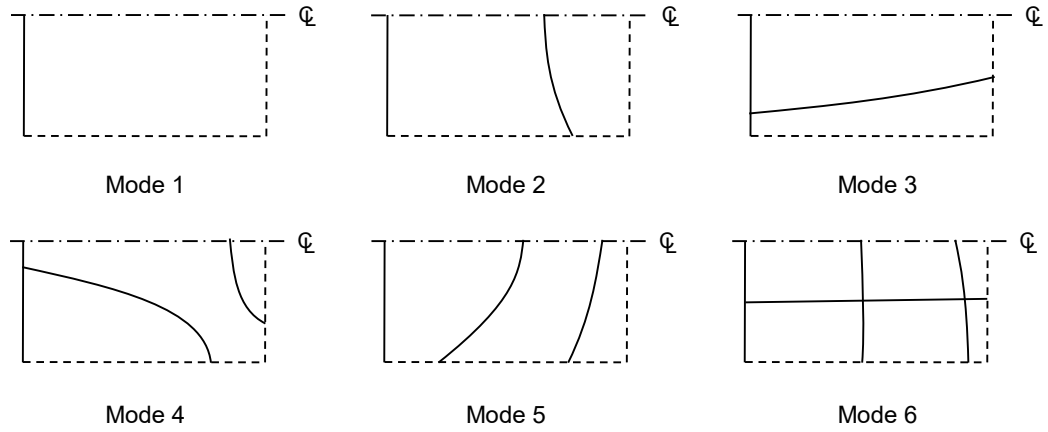


Figure 4.2.3
Vibration mode shapes
(Case B)

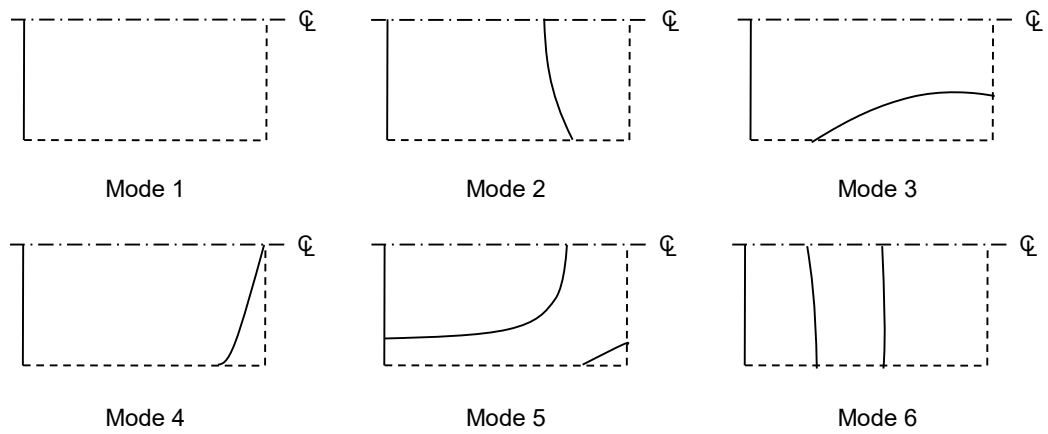


Table 4.2.1 Natural frequencies in Hz obtained using shell elements; symmetric vibration modes (Case A)

Mode Number		1	2	3	4	5	6
Reference		0.421	2.582	3.306	6.555	7.381	11.402
Element type	Number of elements						
TRIA-3	64	0.418	2.639	3.369	6.951	7.974	13.032
QUAD-4	8x4	0.418	2.615	3.337	6.752	7.905	12.681
TRIA-6	16	0.418	2.637	3.290	6.748	7.809	12.636
QUAD-8	4x2	0.419	2.569	3.281	6.566	7.497	12.129



Table 4.2.2 Natural frequencies in Hz obtained using solid elements; symmetric vibration modes (Case A)

Mode Number		1	2	3	4	5	6
Reference		0.421	2.582	3.306	6.555	7.381	11.402
Element type	Number of elements						
HEXA-8	8x4x1	0.419	2.656	3.353	6.747	8.251	12.654
PENTA-15	16	0.436	2.909	4.043	9.401	10.253	21.691
HEXA-20	4x2x1	0.431	2.664	3.426	7.617	7.885	14.343

Table 4.2.3 Natural frequencies in Hz obtained using shell elements; anti-symmetric vibration modes (Case B)

Mode Number		1	2	3	4	5	6
Reference		1.029	3.753	7.730	8.561	N/A	N/A
Element type	Number of elements						
TRIA-3	64	1.017	3.788	8.201	9.111	12.246	17.365
QUAD-4	8x4	1.026	3.807	8.220	9.194	11.974	18.004
TRIA-6	16	1.022	3.744	7.849	8.732	11.292	16.426
QUAD-8	4x2	1.024	3.730	7.625	8.606	11.167	16.807

Table 4.2.4 Natural frequencies in Hz obtained using solid elements; anti-symmetric vibration modes (Case B)

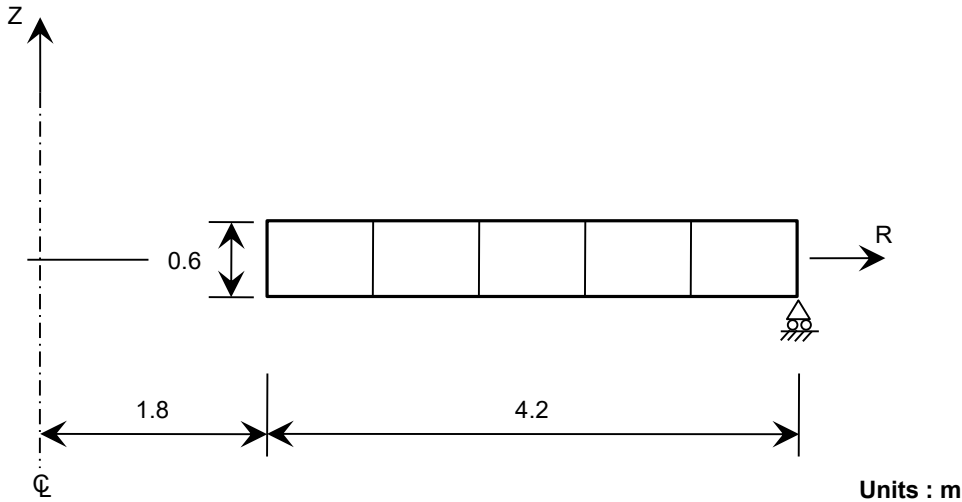
Mode Number		1	2	3	4	5	6
Reference		1.029	3.753	7.730	8.561	N/A	N/A
Element type	Number of elements						
HEXA-8	8x4x1	1.029	3.840	8.356	9.416	11.977	18.248
PENTA-15	16	1.197	4.880	10.618	13.128	22.087	27.706
HEXA-20	4x2x1	1.043	3.830	8.237	9.102	14.733	17.537

4.3 Simply Supported Thick Annular Plate (Axisymmetric)

REFERENCE	NAFEMS ¹
ELEMENTS	Axisymmetric elements, solid elements
MODEL FILENAME	Dynamics03(Axisym).fea Dynamics03(SOLID).fea

A simply-supported thick annular plate problem is depicted in Figure 4.3.1. As an axisymmetric problem, the annular plate is modeled using axisymmetric elements and solid elements with the boundary condition to enforce axisymmetry. The natural frequencies of the lowest modes of which the vibration mode shapes are depicted in Figure 4.3.2 are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.3.1
Simply-supported
axisymmetric annular
plate model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$
Section property	Thickness	$t = 0.6 \text{ m}$



Figure 4.3.2
Vibration mode shapes

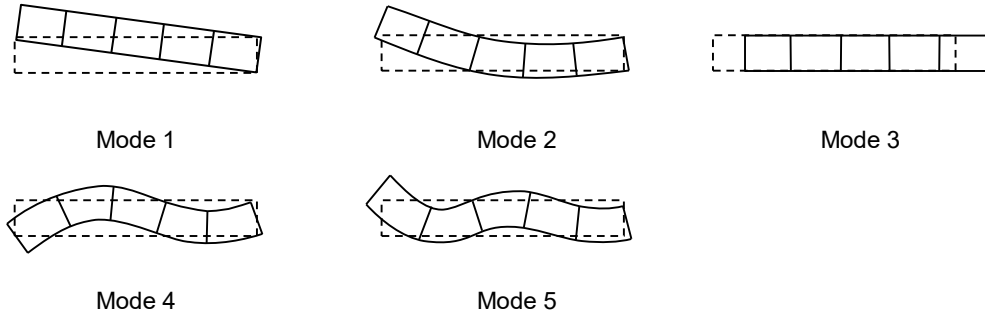


Table 4.3.1 Natural frequencies in Hz obtained using axisymmetric elements

Mode Number		1	2	3	4	5
Reference		18.583	140.15	224.16	358.29	629.19
Element type	Number of elements					
QUADX-4	5	18.569	147.47	223.63	441.20	695.50
TRIAX-3	10	20.428	223.39	291.56	684.85	731.18
QUADX-8	5	18.582	140.56	224.18	374.05	686.04
TRIAX-6	10	18.590	140.90	224.16	379.00	688.48

Table 4.3.2 Natural frequencies in Hz obtained using solid elements

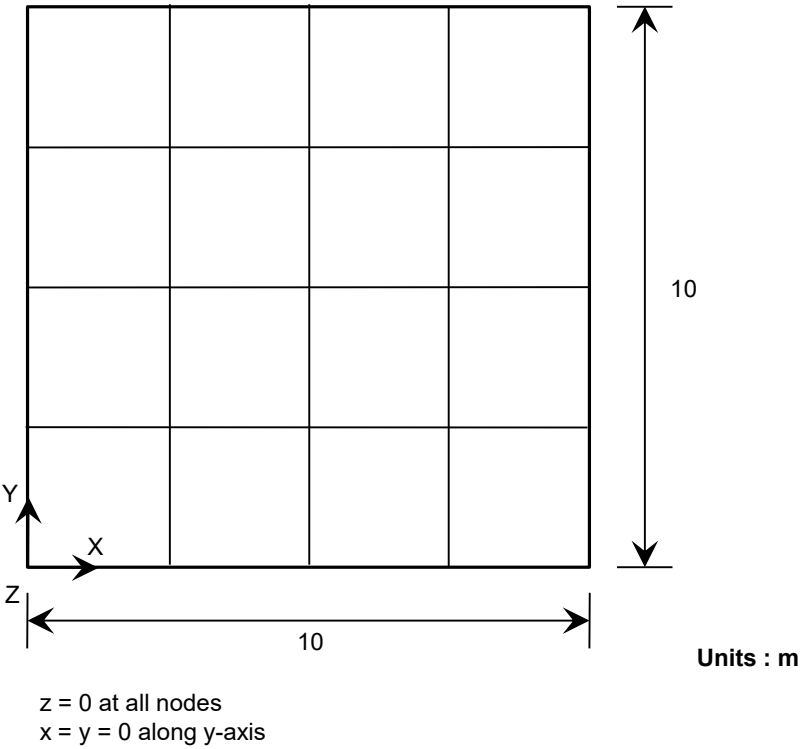
Mode Number		1	2	3	4	5
Reference		18.583	140.15	224.16	358.29	629.19
Element type	Number of elements					
TETRA-4	565	20.910	159.43	224.52	413.64	688.22
PENTA-6	120	18.951	141.08	224.55	366.24	648.70
HEXA-8	15x1x4	18.600	139.20	224.22	362.81	645.35
TETRA-10	59	18.893	139.87	224.60	368.30	664.11
PENTA-15	10	18.658	135.18	224.06	344.42	598.98
HEXA-20	5x1x1	18.389	133.28	223.28	338.86	590.50

4.4 Cantilevered Square Plane Stress

REFERENCE	NAFEMS ¹
ELEMENTS	Plane stress elements
MODEL FILENAME	Dynamics04.fea

Figure 4.4.1 shows a cantilevered square plane stress problem. Plane stress elements are employed to obtain the lowest vibration frequencies. The results are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.4.1
Cantilevered square
plane stress model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$



Figure 4.4.2
Vibration mode shapes

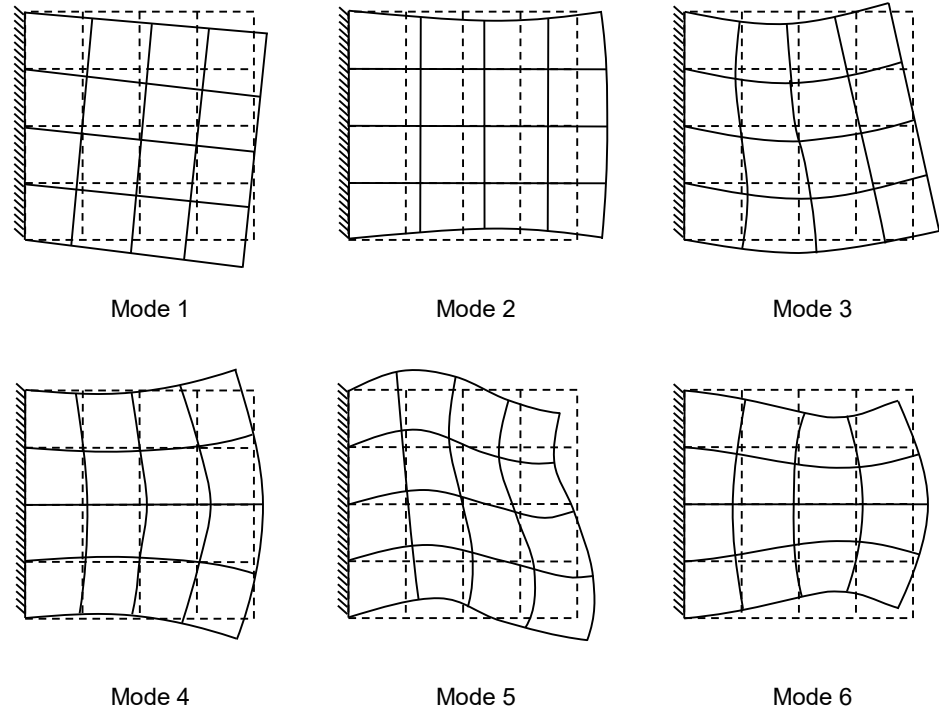


Table 4.4.1 Natural frequencies in Hz obtained using plane stress elements

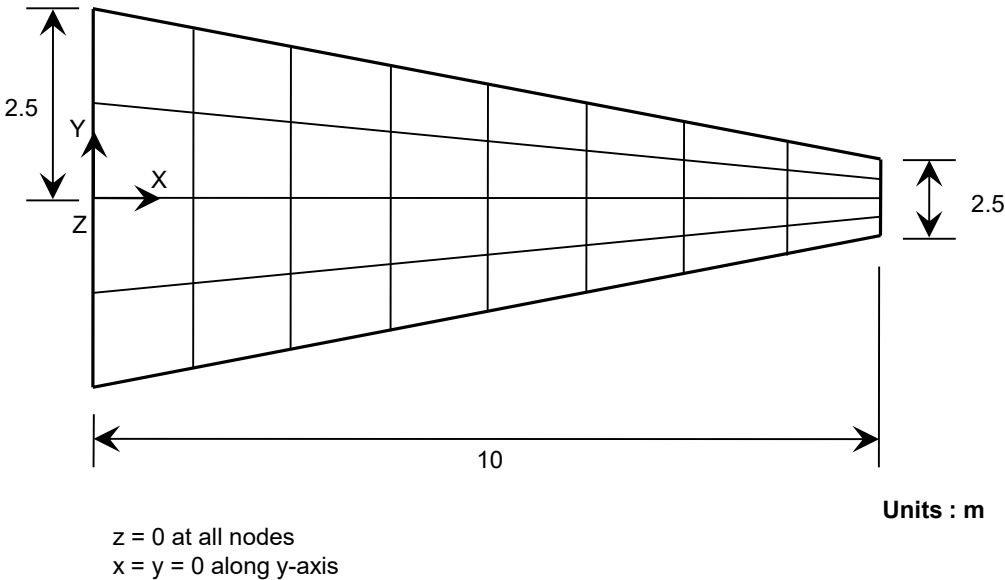
Mode Number		1	2	3	4	5	6
Reference		52.404	125.69	140.78	222.54	241.41	255.74
Element type	Number of elements						
TRIAX-3	128	53.549	125.89	142.19	226.83	243.51	259.46
QUADX-4	8x8	52.426	125.59	139.48	214.51	239.81	252.03
TRIAX-6	32	52.646	125.87	141.76	225.67	243.90	257.64
QUADX-8	4x4	52.603	125.85	141.41	224.45	243.00	256.67

4.5 Cantilevered Tapered Membrane

REFERENCE	NAFEMS ²
ELEMENTS	Plane stress elements
MODEL FILENAME	Dynamics05.fea

Figure 4.5.1 shows a cantilevered tapered membrane problem. Plane stress elements are employed to obtain the lowest vibration frequencies. The results are compared with the reference values provided in the NAFEMS benchmarks.

Figure 4.5.1
Cantilevered tapered
membrane model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$



Figure 4.5.2
Vibration mode shapes

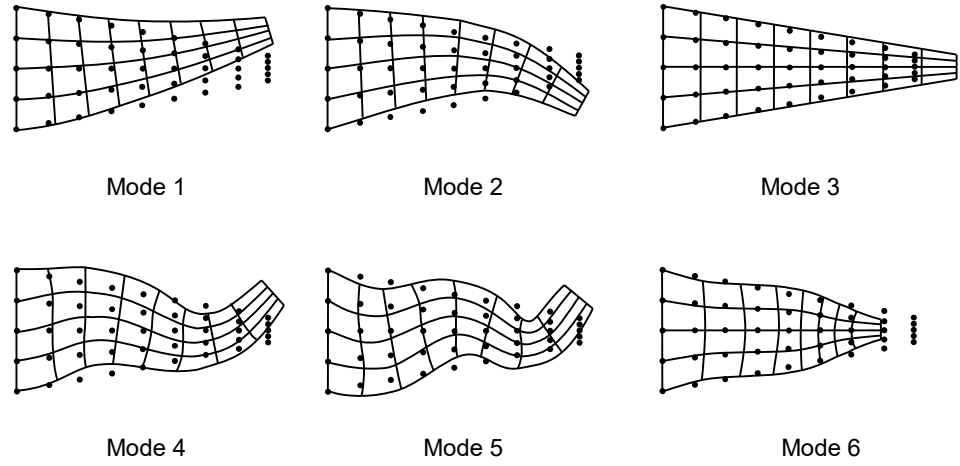


Table 4.5.1 Natural frequencies in Hz obtained using plane stress elements

Mode Number		1	2	3	4	5	6
Reference		44.623	130.03	162.70	246.05	379.90	391.44
Element type	Number of elements						
TRIA-3	128	45.643	134.53	162.92	258.37	393.48	404.95
QUAD-4	8x8	44.647	131.04	162.80	250.33	391.54	393.10
TRIA-6	32	44.645	130.19	162.72	247.05	383.47	391.58
QUAD-8	4x4	44.632	130.11	162.71	246.48	381.58	391.53

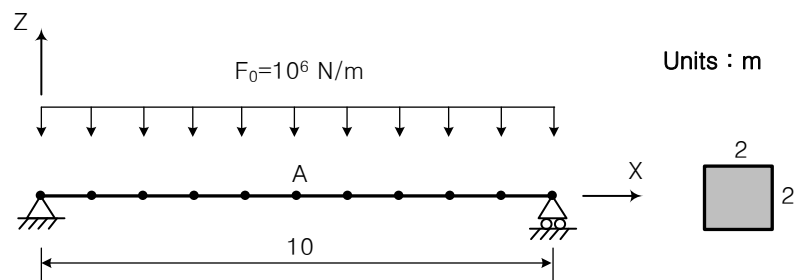


4.6 Deep Simply Supported Beam

REFERENCE	NAFEMS ³
ELEMENTS	Beam elements, solid elements
MODEL FILENAME	Dynamics06(Direct).fea Dynamics06(Modal).fea

A dynamic system consisting of a simply supported beam is shown in Figure 4.6.1. Two types of time-variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings, (2) a transient step load. Each load is applied in the transverse direction of the beam and uniformly distributed spatially. The peak displacement and stress are obtained in the time domain. The results are obtained at the mid-span using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.6.1 - 4.6.2. The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.6.1
Simply supported deep
beam model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$
Section property	Rectangular cross-section	$2 \text{ m} \times 2 \text{ m}$
Forcing functions	Periodic	$F = F_0 (\sin \omega t - \sin 3\omega t)$ (where $\omega = 2\pi f$, $f = 20 \text{ Hz}$)
	Transient	$F = F_0, t > 0$

* Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.



Table 4.6.1 Peak responses of beam subjected to periodic forcing function

		Peak u_z^A [mm]		Peak σ^A [MPa]	
Reference		0.951		17.1	
Element type	Number of elements	Direct	Modal	Direct	Modal
BEAM-2	10	0.955	0.955	17.5	17.4
HEXA-8	10	0.962	0.962	17.5	17.3
PENTA-6	10x4	0.964	0.965	17.3	18.1
HEXA-20	5	0.945	0.944	17.1	17.0
PENTA-15	5x4	0.945	0.945	16.9	16.9
PYRAM-13	5x6	0.944	0.945	17.5	17.5

Table 4.6.2 Peak responses of beam subjected to transient step load

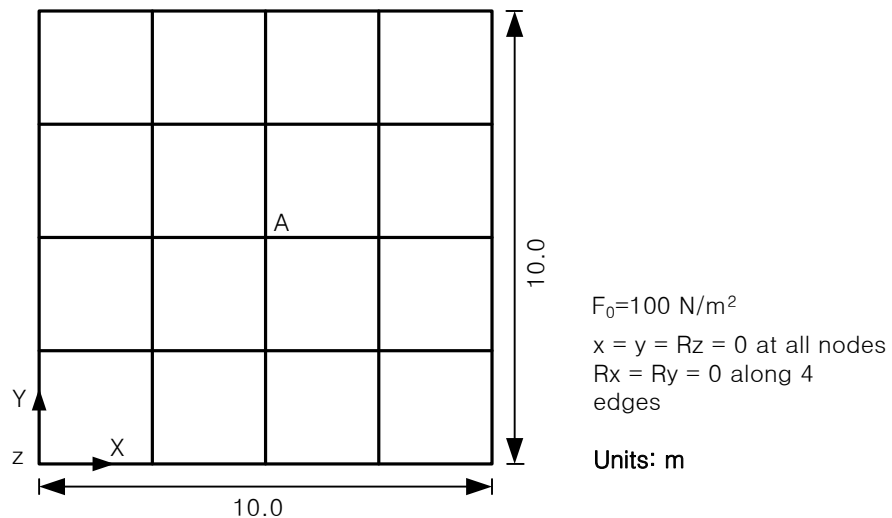
		Peak u_z^A [mm]		Peak time [sec]		Peak σ^A [MPa]		Static u_z^A [mm]	
Reference		1.043		0.0117		18.76		0.538	
Element type	Number of elements	Direct	Modal	Direct	Modal	Direct	Modal	Direct	Modal
BEAM-2	10	1.044	1.043	0.0117	0.0116	18.51	18.51	0.537	0.537
HEXA-8	10	1.012	1.1012	0.0116	0.0116	18.03	18.03	0.521	0.521
PENTA-6	10x4	0.946	0.945	0.0112	0.0112	17.40	17.31	0.487	0.487
HEXA-20	5	1.019	1.017	0.117	0.0116	18.06	17.96	0.524	0.525
PENTA-15	5x4	1.023	1.022	0.117	0.0116	18.05	18.04	0.527	0.527
PYRAM-13	5x6	1.014	1.013	0.117	0.0116	17.99	17.88	0.523	0.522

4.7 Simply Supported Thin Square Plate

REFERENCE	NAFEMS ³
ELEMENTS	Shell elements
MODEL FILENAME	Dynamics07(Direct).fea Dynamics07(Modal).fea

A dynamic system consisting of a thin square cantilever plate is shown in Figure 4.7.1. Two types of time variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings, (2) a transient step load. Each load is uniformly distributed spatially and applied in the transverse direction of the plate. The peak displacement and stress are obtained either in the frequency domain or the time domain. The results are obtained at the center point using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.7.1-4.7.2. The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.7.1
Simply supported thin
plate model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$
	Mass proportional damping	$\alpha = 0.229 \text{ sec}^{-1}$
	Stiffness proportional damping	$\beta = 1.339 \times 10^{-3} \text{ sec}$
	Modal damping	$\xi = 0.02$
Section property	Thickness	$t = 0.05 \text{ m}$
Forcing functions	(1) Periodic	$F = F_0 (\sin \omega t - \sin 3\omega t)$ (where $\omega = 2\pi f$, $f = 20 \text{ Hz}$)
	(2) Transient	$F = F_0, t > 0$

* Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.

Table 4.7.1 Peak responses of thin plate subjected to periodic forcing function



		Peak u_z^A [mm]		Peak σ^A [MPa]	
Reference		2.863		2.018	
Element type	Number of elements	Direct	Modal	Direct	Modal
QUAD-4	8x8	2.913	2.916	2.072	2.077
TRIA-3	128	2.881	2.883	2.068	2.074
QUAD-8	4x4	2.884	2.885	2.274	2.277
TRIA-6	32	2.890	2.892	2.177	2.177

Table 4.7.2 Peak responses of thin plate subjected to transient step load

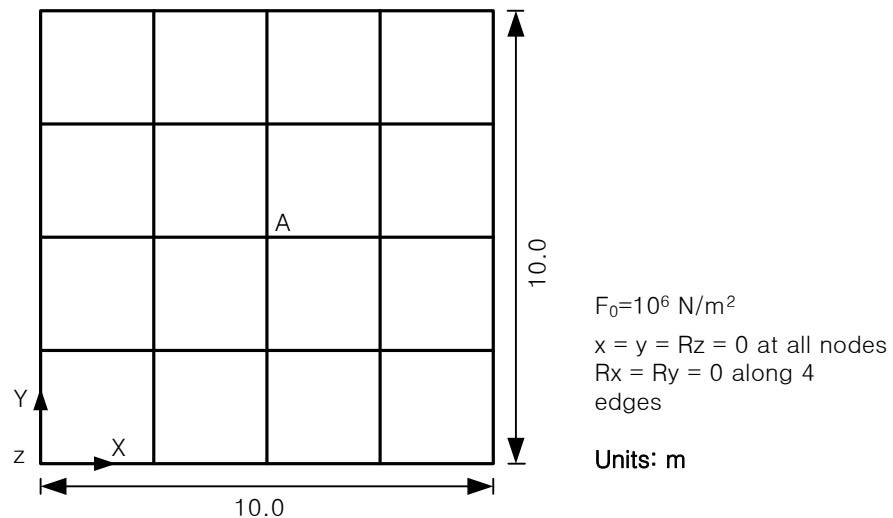
		Peak u_z^A [mm]		Peak time [sec]		Peak σ^A [MPa]		Static u_z^A [mm]	
Reference		3.523		0.210		2.484		1.817	
Element type	Number of elements	Direct	Modal	Direct	Modal	Direct	Modal	Direct	Modal
QUAD-4	8x8	3.474	3.487	0.210	0.211	2.376	2.425	1.770	1.770
TRIA-3	128	3.342	3.354	0.206	0.207	2.306	2.349	1.703	1.703
QUAD-8	4x4	3.472	3.472	0.214	0.216	2.522	2.541	1.774	1.774
TRIA-6	32	3.485	3.473	0.210	0.207	2.355	2.321	1.786	1.786

4.8 Simply Supported Thick Square Plate

REFERENCE	NAFEMS ³
ELEMENTS	Shell elements, solid elements
MODEL FILENAME	Dynamics08(Direct).fea Dynamics08(Modal).fea

A dynamic system consisting of a thick square cantilever plate is shown in Figure 4.8.1. Two types of time-variation of the load are considered; (1) a periodic forcing function composed of two harmonic loadings and (2) a transient step load. Each load is uniformly distributed spatially and applied in the transverse direction of the plate. The peak displacement and stress are obtained in the time domain. The results are obtained at the center point using both modal superposition and direct methods. The responses obtained using various elements corresponding to each load type are summarized in Tables 4.8.1-4.8.2. The results from the NAFEMS benchmark problems are taken as reference.

Figure 4.8.1
Simply supported thick
plate model



Material data	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 78.4532 \text{ kN/m}^3$
	Mass proportional damping	$\alpha = 0.229 \text{ sec}^{-1}$
	Stiffness proportional damping	$\beta = 1.339 \times 10^{-3} \text{ sec}$
	Modal damping	$\xi = 0.02$
Section property	Thickness	$t = 1.0 \text{ m}$
Forcing functions	(1) Periodic	$F = F_0 (\sin \omega t - \sin 3\omega t)$ (where $\omega = 2\pi f$, $f = 20 \text{ Hz}$)
	(2) Transient	$F = F_0, t > 0$

* Rayleigh damping coefficients, α and β are chosen to give 0.02 damping in the dominant first mode.

Table 4.8.1 Peak responses of thin plate subjected to periodic forcing function



		Peak u_z^A [mm]		Peak σ^A [MPa]	
Reference		4.929		67.67	
Element type	Number of elements	Direct	Modal	Direct	Modal
QUAD-4	8x8	5.294	5.298	72.35	72.46
TRIA-3	128	5.419	5.422	73.26	73.32
QUAD-8	4x4	5.058	5.061	72.13	72.19
TRIA-6	32	5.022	5.024	70.91	70.98
HEXA-8	96	5.344	53.37	70.19	70.87
HEXA-20	48	5.048	5.050	76.49	76.57
PENTA-15	64	5.163	5.165	73.06	73.12

Table 4.8.2 Peak responses of thin plate subjected to transient step load

		Peak u_z^A [mm]		Peak time [sec]		Peak σ^A [MPa]		Static u_z^A [mm]	
Reference		4.524		0.0108		62.11		2.333	
Element type	Number of elements	<i>Direct</i>	<i>Modal</i>	<i>Direct</i>	<i>Modal</i>	<i>Direct</i>	<i>Modal</i>	<i>Direct</i>	<i>Modal</i>
QUAD-4	8x8	4.563	4.547	0.010 7	0.010 5	58.06	57.40	2.339	2.339
TRIA-3	128	4.388	4.372	0.010 5	0.105	55.66	54.84	2.249	2.249
QUAD-8	4x4	4.827	4.845	0.0110	0.010 9	66.71	67.81	2.423	2.423
TRIA-6	32	4.773	4.778	0.010 9	0.010 8	63.68	64.06	2.422	2.421
HEXA-8	96	4.505	4.489	0.010 8	0.010 9	56.39	55.80	2.310	2.310
HEXA-20	48	4.467	4.458	0.010 7	0.010 6	62.85	62.35	2.283	2.283
PENTA-15	64	4.354	4.343	0.010 6	0.010 3	57.57	57.39	2.229	2.229

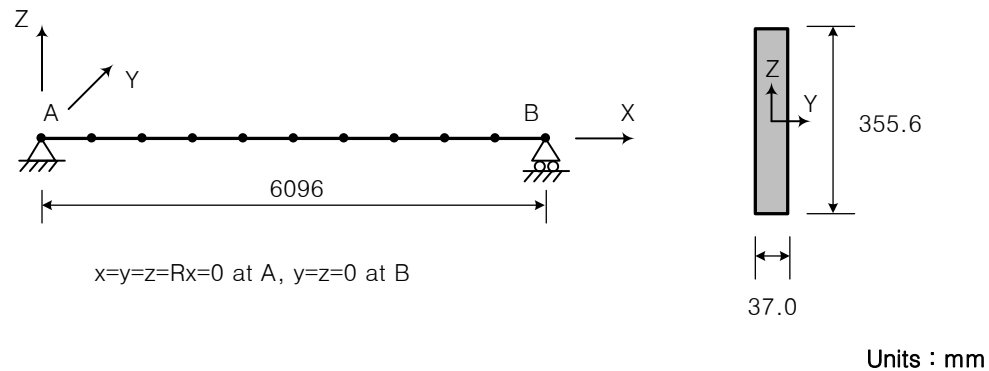


4.9 Response Spectrum of a Simply Supported Beam

REFERENCE	Biggs, J. M. ⁴
ELEMENTS	Beam elements, shell elements, solid elements
MODEL FILENAME	Dynamics09.fea

Figure 4.9.1 shows a simply-supported beam problem with a rectangular cross section. The response spectrum defined in Table 4.9.1 is applied in the vertical direction of the beam at its ends. Response spectrum analyses are carried out with various types of elements. Damping is assumed to be zero. The maximum displacement, bending moment and stress at the mid-span are obtained using the square root of the sum of squares (SRSS) summation method.

Figure 4.9.1
Simply supported beam
model



Material data	Young's modulus	$E = 206.8 \text{ GPa}$
	Unit Weight	$\gamma = 1027.05046 \text{ kN/m}^3$
Section property	Rectangular cross-section	$2 \text{ m} \times 2 \text{ m}$

*Table 4.9.1 Response spectra definition(unit : m)*

Frequency [Hz]		5.000	6.000	6.098	7.000	8.000
Period [sec]		0.2000	0.1667	0.1640	0.1429	0.1250
Type (scale factor)	Displacement(1.0)	0.0199	0.0115	0.0110	0.0072	0.0056
	Velocity(1.0)	0.6248	0.4339	0.4201	0.3188	0.2837
	Acceleration(0.5)	39.258	32.716	32.190	28.042	28.521

Table 4.9.2 Response spectrum analysis results obtained using beam elements

Result at mid-span		Displacement [mm]	Stress [MPa]	Moment $\times 10^5$ [Nm]
Reference		14.2	140.4	1.095
Element type	Spectra type			
BEAM	Displacement	14.2	138.4	1.079
	Velocity	14.1	138.1	1.077
	Acceleration	14.1	138.1	1.077

*Table 4.9.3 Response spectrum analysis results obtained using shell elements*

Result at mid-span		Displacement [mm]	Stress [MPa]
Reference		14.2	140.4
Element type	Spectra type		
QUAD-4	Displacement	13.8	132.4
	Velocity	13.7	132.0
	Acceleration	13.7	132.0
TRIA-3	Displacement	13.9	134.6
	Velocity	13.9	134.2
	Acceleration	13.9	134.2
QUAD-8	Displacement	14.2	138.4
	Velocity	14.1	138.1
	Acceleration	14.1	138.1
TRIA-6	Displacement	14.2	138.8
	Velocity	14.2	138.6
	Acceleration	14.2	138.6

Table 4.9.4 Response spectrum analysis results obtained using solid elements

Result at mid-span		Displacement [mm]	Stress [MPa]
Reference		14.2	140.4
Element type	Spectra type		
HEXA-8	Displacement	13.7	132.3
	Velocity	13.7	131.9
	Acceleration	13.7	131.9
HEXA-20	Displacement	14.1	138.3
	Velocity	14.1	138.0
	Acceleration	14.1	138.0
PENTA-15	Displacement	14.1	137.6
	Velocity	14.1	137.3
	Acceleration	14.1	137.3

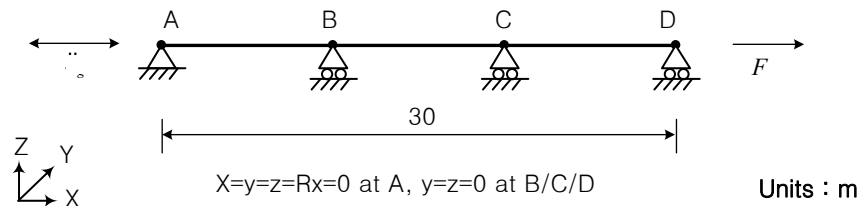


4.10 Linear Dynamic Analyses of a Truss

REFERENCE	Chopra, A. K. ⁵
ELEMENTS	Truss elements, plane stress elements, solid elements
MODEL FILENAME	Dynamics10.fea

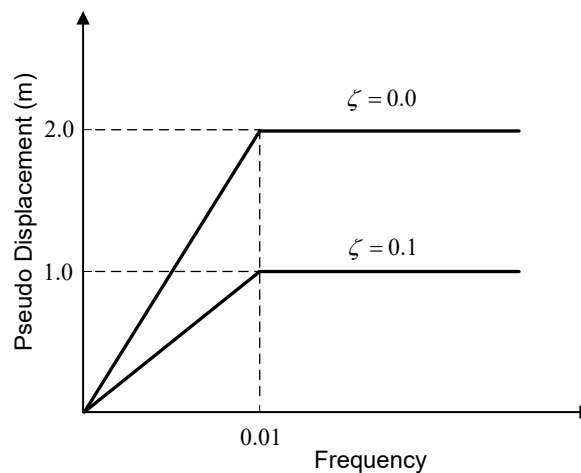
A one-dimensional truss model with three members is depicted in Figure 4.10.1. Truss, plane stress and solid elements are used to model the problem. The results are obtained at the point D using modal methods including transient analysis and response spectrum. Comparison is made with the closed form solutions.

Figure 4.10.1
Simply supported beam
model



Material data	Young's modulus	$E = 5 \text{ Pa}$
	Unit weight	$\gamma = 0.108961688 \text{ N/m}^3$
Section property	Cross-section Area	$A = 2.0 \text{ m}^2$
Analysis condition	Modal transient with tip load	$F = 10 \text{ N}$, 10% damping
	Modal transient with base acceleration	$x_R = 1.0 \text{ m/sec}^2$, 10% damping
	Response spectrum	displacement spectra, 2% damping

Figure 4.10.2
Displacement response
spectra



*Table 4.10.1 Displacement and acceleration at point D using modal transient analysis with tip load*

Result type	Displacement [m]			Acceleration [m/sec ²]		
Time step [sec]	0.1	0.2	0.3	0.1	0.2	0.3
Reference	0.4387	1.686	3.598	81.42	66.71	48.06
Element type						
TRUSS-2	0.4387	1.686	3.598	81.42	66.71	48.06
QUAD-4	0.4387	1.686	3.598	81.42	66.71	48.06
HEXA-8	0.4387	1.686	3.598	81.42	66.71	48.06

Table 4.10.2 Total displacement at point D using modal transient analysis with base acceleration

Result type	Displacement [m]		
Time step [sec]	0.1	0.2	0.3
Reference	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
Element type			
TRUSS-2	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
QUAD-4	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}
HEXA-8	0.244×10^{-4}	1.965×10^{-4}	6.692×10^{-4}

Table 4.10.3 Peak displacement at point D using response spectrum analysis with 2% modal damping ratio

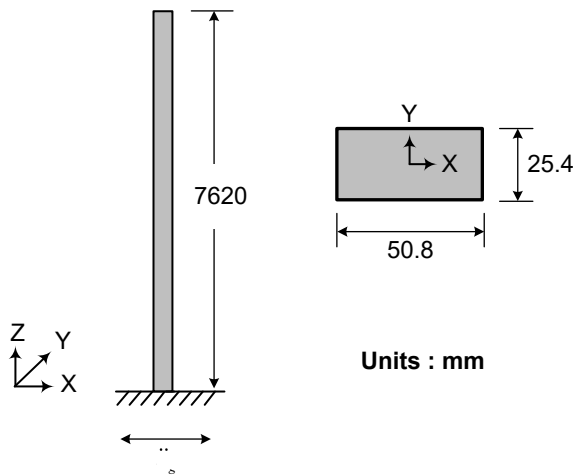
Result type	Displacement [m]				
Combination method	ABS	SRSS	TENP	NRL	CQC
Reference	2.902	2.248	2.248	2.767	2.246
Element type	2.902	2.248	2.248	2.767	2.246
TRUSS-2	2.902	2.248	2.248	2.767	2.246
QUAD-4	2.902	2.248	2.248	2.767	2.246
HEXA-8	2.902	2.248	2.248	2.767	2.246

4.11 Cantilever Subjected to Earthquake Motion

REFERENCE	Hiber, H. M. et al. ⁶ and Hurty, W.C. et al. ⁷
ELEMENTS	Beam elements
MODEL FILENAME	Dynamics11.fea

Figure 4.11.1 shows a vertical cantilever beam model. Seismic analyses are carried out in which the anchor point of the beam model is subjected to a prescribed motion. The time history of acceleration of the El Centro N-S as shown in Figure 4.11.2 is applied in the horizontal direction. The transient response of the structure is obtained using the direct and modal superposition methods. The maximum displacement and velocity at the top location is summarized in Table 4.11.1. Response spectrum analyses are also conducted based on the response spectra shown in Figures 4.11.3 and 4.11.4 using the ABS and SRSS combination methods.

Figure 4.11.1
Simply supported beam
model



Material data	Young's modulus	$E = 206.8 \text{ GPa}$
	Unit Weight	$\gamma = 78.4532 \text{ kN/m}^3$
Section property	Rectangular cross-section	$50.8 \text{ mm} \times 25.4 \text{ mm}$



Figure 4.11.2
El Centro N-S
acceleration history

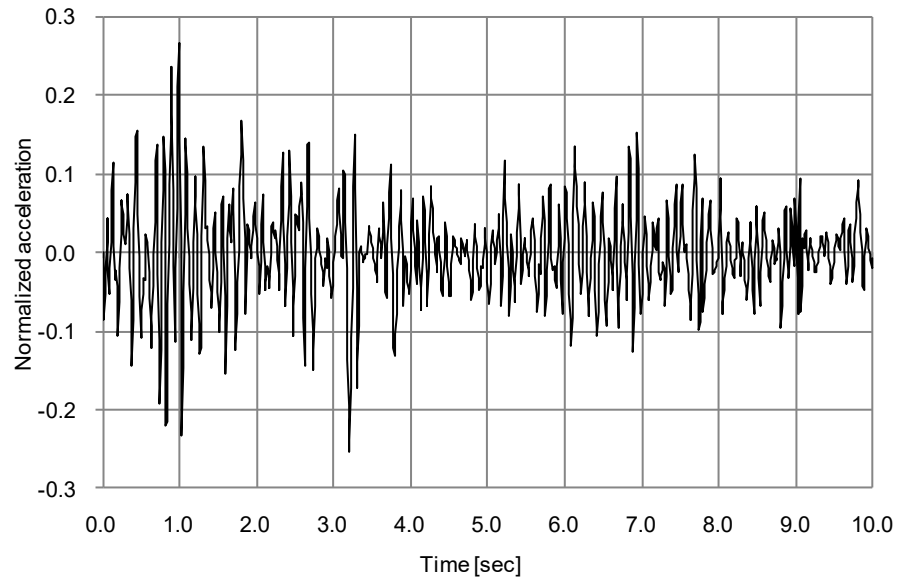


Figure 4.11.3
Displacement spectra
for the period range
0.03~10 sec

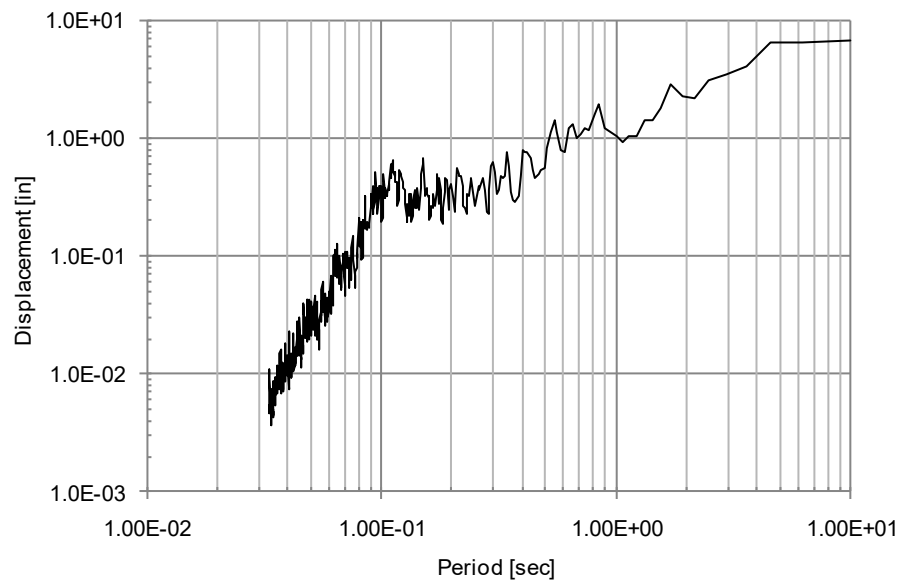




Table 4.11.1 Maximum displacement and velocity at the top of the column provided by transient analysis

Result type		Displacement [mm]	Velocity [m/sec]
Reference		59.2	0.508
Analysis type	Number of elements		
Direct transient	10	58.9	0.411
	20	58.9	0.410
	50	58.9	0.410
Modal transient	10	59.2	0.433
	20	59.1	0.434
	50	59.1	0.435

Figure 4.11.4
Velocity spectra for the
period range 0.03~10
sec

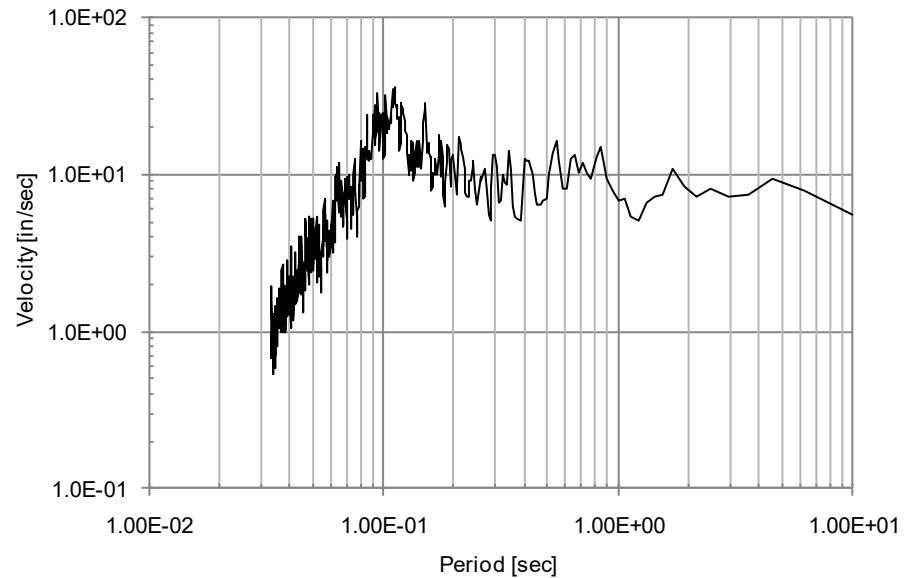




Table 4.11.2 Maximum displacement and velocity at the top of the column provided by response spectrum analysis

Result type			Displacement [mm]	Velocity [m/sec]
Reference			59.2	0.508
Number of elements	Spectrum type	Combination method		
10	Displacement	ABS	67.2	0.639
		SRSS	57.0	0.392
	Velocity	ABS	70.8	0.640
		SRSS	61.0	0.395
20	Displacement	ABS	67.2	0.638
		SRSS	57.0	0.392
	Velocity	ABS	70.8	0.639
		SRSS	61.0	0.395
50	Displacement	ABS	67.2	0.639
		SRSS	57.0	0.392
	Velocity	ABS	70.8	0.639
		SRSS	61.0	0.395

4.12 Rayleigh Wave Velocity

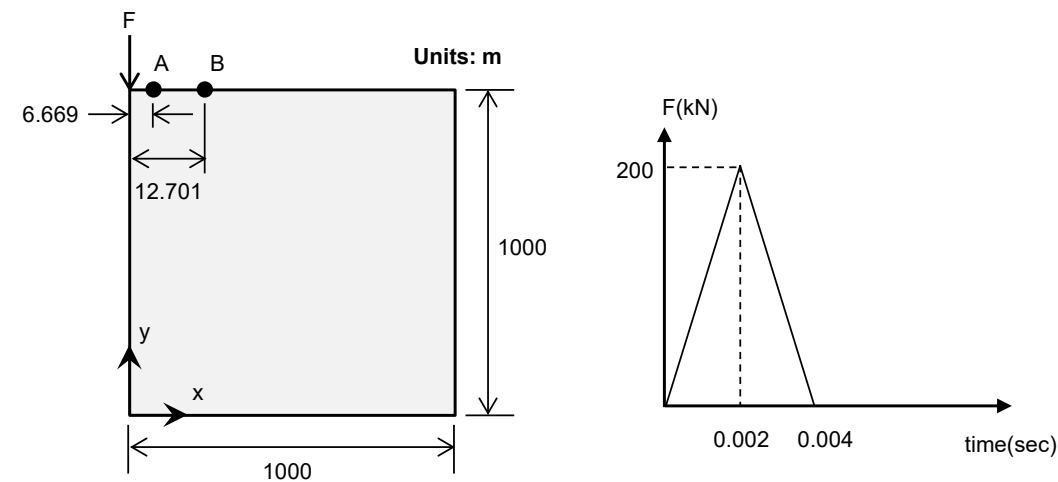
REFERENCE	Knopoff, L. ^{8,9}
ELEMENTS	Axisymmetric elements
MODEL FILENAME	Dynamics12.fea

Figure 4.12.1 shows an axisymmetric model setup for the measurement of the Rayleigh wave velocity. Concentrated load is applied on the top surface in the form of triangular pulse. Direct and modal linear time history analyses are carried out to measure the peak time of Rayleigh wave propagation at point A and B. The time lag and the distance between A and B are used to compute the wave velocity. The calculated Rayleigh wave velocities are compared with theoretical value:

$$V_R = 0.54 \sqrt{\frac{(1-\nu)Eg}{(1+\nu)(1-2\nu)\gamma}}$$

where E is the Young's modulus, ν is the Poisson's ratio, γ is weight density and g is the gravitational acceleration.

Figure 4.12.1
Axisymmetric soil
model and pulse profile



Material data	Young's modulus	$E = 750000 \text{ N/m}^2$
	Poisson's ratio	$\nu = 0.3$
	Unit Weight	$\gamma = 20000 \text{ N/m}^3$
Section property	Thickness	$t = 1.0 \text{ m}$



Figure 4.12.2
Vertical displacements
vs. time at point A and
B obtained using direct
time history analysis

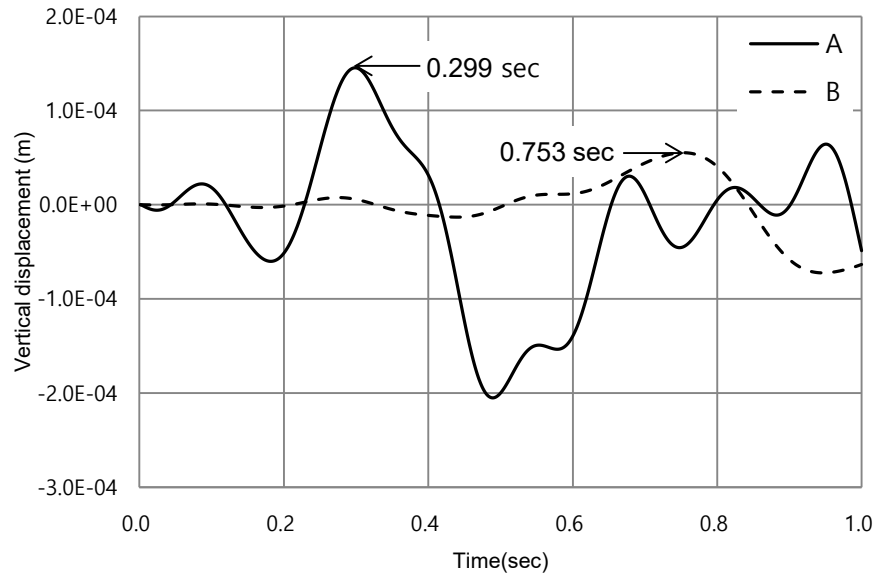


Figure 4.12.3
Vertical displacements
vs. time at point A and
B obtained using modal
time history analysis

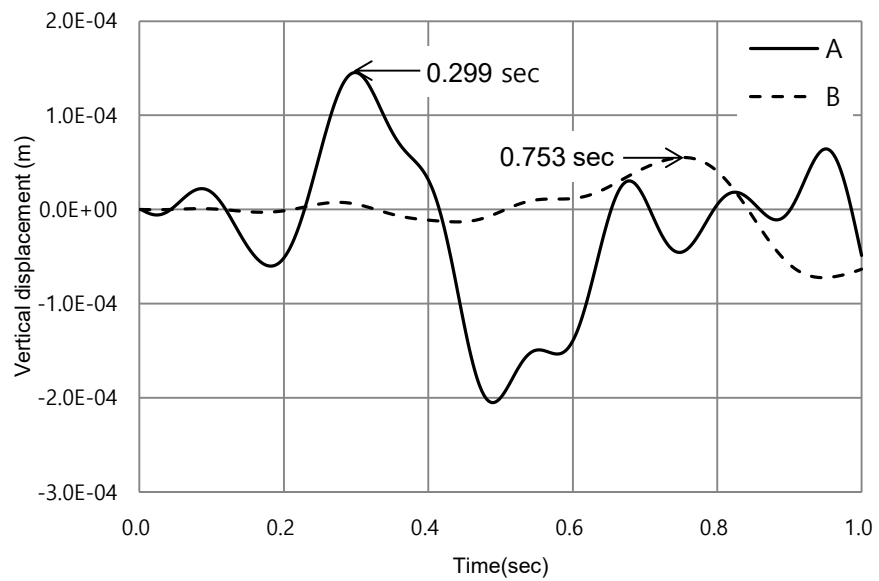


Table 4.12.1 Rayleigh wave velocity between AB

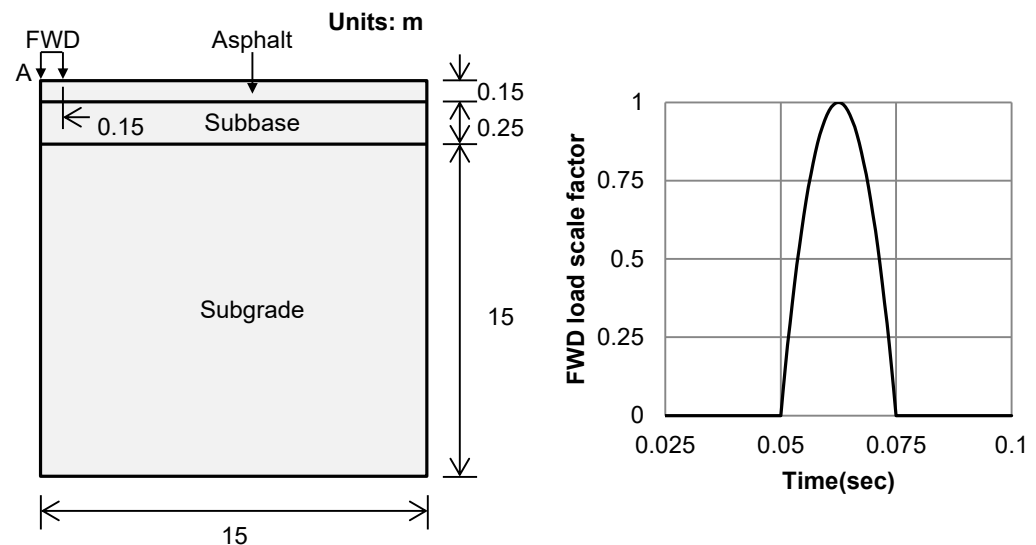
Result type	Velocity [m/sec]
Reference	12.01
Direct Time History	13.28
Modal Time History	13.28

4.13 FWD Load on Pavement

REFERENCE	Al-Khoury, R. et al. ¹⁰
ELEMENTS	Axisymmetric element
MODEL FILENAME	Dynamics13.fea

Figure 4.13.1 shows the axisymmetric model to obtain the transient response of layered pavement subjected to FWD (falling weight deflectometer) load pulse. The pavement is a three layered media containing asphalt, subbase and subgrade. The magnitude of FWD is 50kN and the radius of FWD is 0.15m from the axis of symmetry. Direct and modal linear time history analyses are carried out using axisymmetric elements to obtain the vertical displacement at point A. A semi-analytical solution computed by the spectral element method by adopted for comparison.

Figure 4.13.1
Axisymmetric model of
layered pavement and
FWD load pulse profile



Material data 1 (Asphalt)	Young's modulus	$E = 1000 \text{ MPa}$
	Poisson's ratio	$\nu = 0.35$
	Unit Weight	$\gamma = 2.3 \text{ kN/m}^3$
Material data 2 (Subbase)	Young's modulus	$E = 200 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit Weight	$\gamma = 2.0 \text{ kN/m}^3$
Material data 3 (Subgrade)	Young's modulus	$E = 100 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit Weight	$\gamma = 1.5 \text{ kN/m}^3$
Section property	Thickness	$t = 1.0 \text{ m}$



Figure 4.13.2
Vertical displacement at
A vs. time obtained
using direct and modal
linear time history
analyses

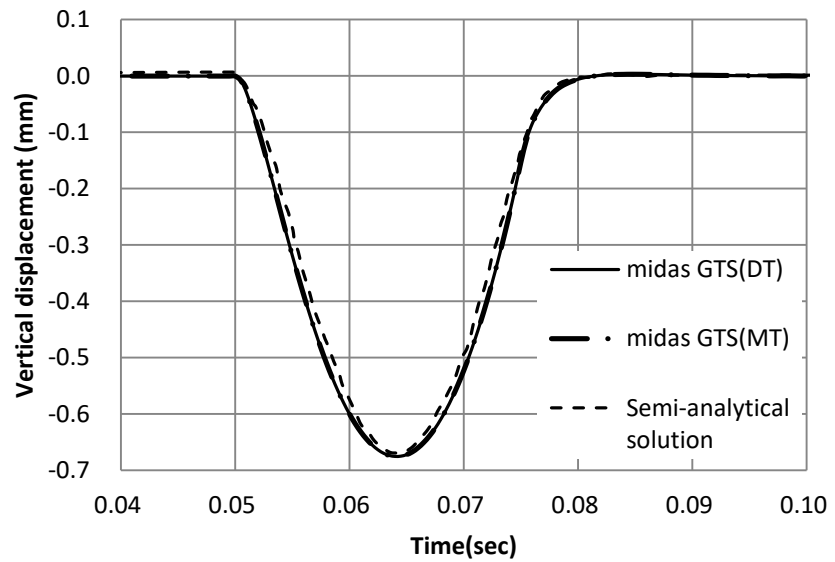


Table 5.1.1 Maximum vertical displacement at the center of symmetry

Result type	Vertical displacement [m/sec]
Reference	0.669
Direct Time History	0.675
Modal Time History	0.675

4.14 Earthquake Response of a Layered Soil Deposit

REFERENCE	Schnabel, P. B. et al. ¹¹
KEYWORDS	free field, equivalent linear
MODEL FILENAME	Dynamic14.lif, Dynamic14.fea

Figure 4.14 shows a 150-ft soil profile consisting of clay and sand overlying a half-space. The material parameters of individual layers are summarized in Tables 4.14.1 ~ 4.14.4. The soil deposit is subjected to controlled motion corresponding to the earthquake time history which had been recorded at Diamond Heights (EW component) during the Loma Prieta earthquake. This motion is normalized to a peak acceleration of 0.1g. Free-field and equivalent linear analyses are carried out to obtain various responses of the soil deposit. The obtained responses are compared with the results included in the reference.

Figure 4.14.1
Soil profile

5 ft	Layer 1 : material 1, property 1
5 ft	Layer 2 : material 2, property 1
10 ft	Layer 3 : material 2, property 1
10 ft	Layer 4 : material 3, property 1
10 ft	Layer 5 : material 4, property 2
10 ft	Layer 6 : material 4, property 2
10 ft	Layer 7 : material 5, property 2
10 ft	Layer 8 : material 5, property 2
10 ft	Layer 9 : material 6, property 1
10 ft	Layer 10 : material 6, property 1
10 ft	Layer 11 : material 7, property 1
10 ft	Layer 12 : material 7, property 1
10 ft	Layer 13 : material 8, property 1
10 ft	Layer 14 : material 9, property 1
10 ft	Layer 15 : material 10, property 1
10 ft	Layer 16 : material 11, property 1
Bed Rock : material 12, property 3	

Table 4.14.1 Material data of soil profile



Material data	Unit Weight (kgf / ft^3)	Maximum shear modulus (kgf / ft^2)	Damping ratio
1	0.125	3882.0	0.05
2	0.125	3144.0	0.05
3	0.125	3503.0	0.05
4	0.125	3882.0	0.05
5	0.125	4697.0	0.05
6	0.130	6823.0	0.05
7	0.130	7913.0	0.05
8	0.130	9084.0	0.05
9	0.130	10335.0	0.05
10	0.130	13081.0	0.05
11	0.140	69565.0	0.01

Table 4.14.2 Strain compatible property 1(sand)

Sand (Seed and Idriss 1970)		Sand (Idriss 1990)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.000	1.0×10^{-6}	0.0024
3.0×10^{-6}	1.000	3.0×10^{-6}	0.0042
1.0×10^{-5}	0.990	1.0×10^{-5}	0.0080
3.0×10^{-5}	0.960	3.0×10^{-5}	0.0140
1.0×10^{-4}	0.850	1.0×10^{-4}	0.0280
3.0×10^{-4}	0.640	3.0×10^{-4}	0.0510
1.0×10^{-3}	0.370	1.0×10^{-3}	0.0980
3.0×10^{-3}	0.180	3.0×10^{-3}	0.1550
1.0×10^{-2}	0.080	1.0×10^{-2}	0.2100
3.0×10^{-2}	0.050	3.0×10^{-2}	0.2500
1.0×10^{-1}	0.035	1.0×10^{-1}	0.2800

*Table 4.14.3 Strain compatible property 2(clay)*

Clay (Seed and Sun 1989)		Clay (Idriss 1990)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.000	1.00×10^{-6}	0.0024
3.0×10^{-6}	1.000	3.00×10^{-6}	0.0042
1.0×10^{-5}	1.000	1.00×10^{-5}	0.0080
3.0×10^{-5}	0.981	3.00×10^{-5}	0.0140
1.0×10^{-4}	0.941	1.00×10^{-4}	0.0280
3.0×10^{-4}	0.847	3.00×10^{-4}	0.0510
1.0×10^{-3}	0.656	1.00×10^{-3}	0.0980
3.0×10^{-3}	0.438	3.00×10^{-3}	0.1550
1.0×10^{-2}	0.238	1.00×10^{-2}	0.2100
3.0×10^{-2}	0.144	3.16×10^{-2}	0.2500
1.0×10^{-1}	0.110	1.00×10^{-1}	0.2800

Table 4.14.4 Strain compatible property 3(rock)

Rock (Idriss)		Rock (Idriss)	
Shear strain	Shear modulus reduction factor	Shear strain	Damping ratio
1.0×10^{-6}	1.0000	1.00×10^{-6}	0.004
3.0×10^{-6}	1.0000	1.00×10^{-5}	0.008
1.0×10^{-5}	0.9875	1.00×10^{-4}	0.015
3.0×10^{-5}	0.9525	1.00×10^{-3}	0.030
1.0×10^{-4}	0.9000	1.00×10^{-2}	0.046
3.0×10^{-4}	0.8100		
1.0×10^{-3}	0.7250		
1.0×10^{-2}	0.5500		



Figure 4.14.2
Controlled acceleration
history corresponding
to Diamond Heights
earthquake (EW
components)

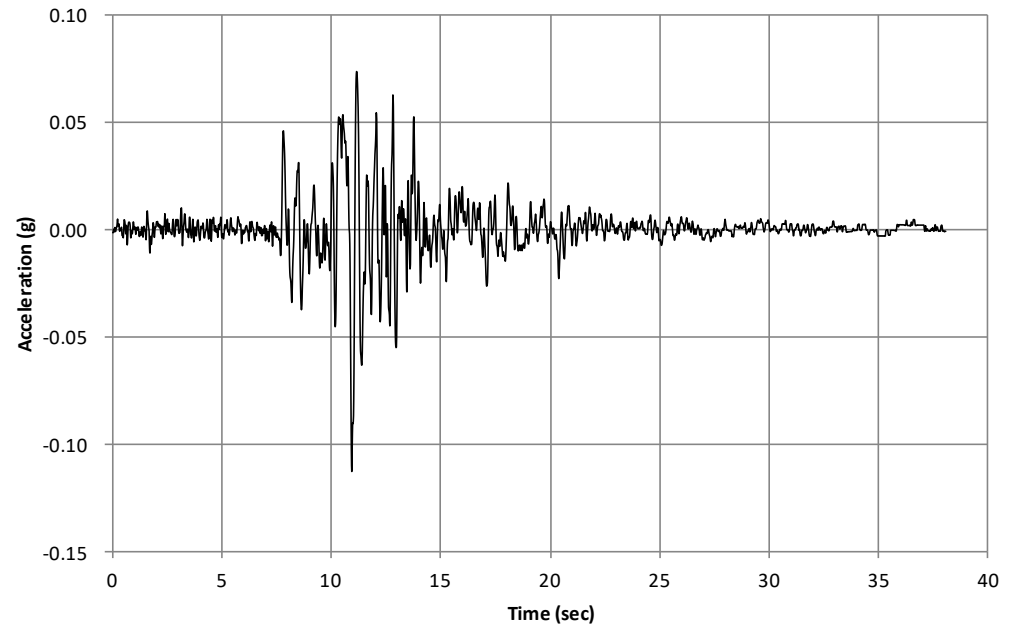


Table 4.14.5 Maximum acceleration with depth

	Free Field	2D Equivalent	Reference
Depth (ft)	Maximum Acceleration(g)	Maximum Acceleration(g)	Maximum Acceleration(g)
0	0.19033	0.19081	0.19037
5	0.18999	0.19035	0.19006
10	0.18866	0.18875	0.18876
20	0.18253	0.18275	0.18258
30	0.17206	0.17248	0.17208
40	0.15946	0.16018	0.15947
50	0.14289	0.14387	0.14288
60	0.12653	0.12742	0.12652
70	0.11042	0.11022	0.11050
80	0.09837	0.09825	0.09840
90	0.08997	0.08955	0.08999
100	0.08266	0.08216	0.08268
110	0.08562	0.08586	0.08559
120	0.08548	0.08568	0.08547
130	0.08201	0.08216	0.08198
140	0.07767	0.07750	0.07769
150 (Within)	0.07614	0.07613	0.07617
150 (Outcrop)	0.09999	-	0.10000



Figure 4.14.3
Acceleration time
history at ground
surface obtained using
free field and
equivalent linear
analyses

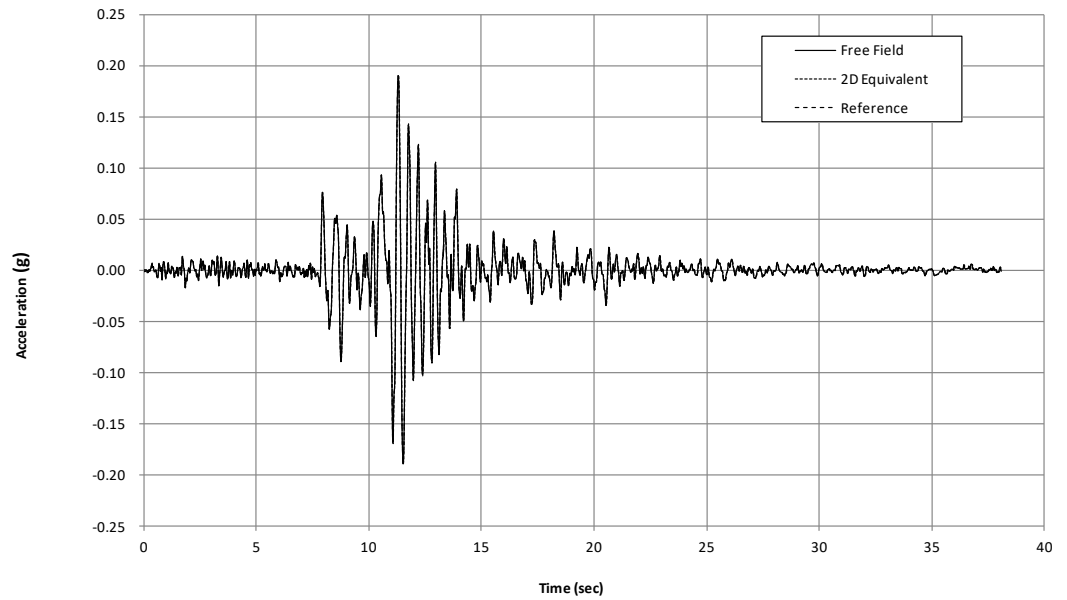


Table 4.14.6 Maximum strain and stress with depth

Depth (ft)	Free Field		2D Equivalent		Reference	
	Maximum strain (%)	Maximum stress (lb / ft ²)	Maximum strain (%)	Maximum stress (lb / ft ²)	Maximum strain (%)	Maximum stress (lb / ft ²)
2.5	0.00154	59.42	0.00153	59.01	0.00154	59.43
7.5	0.00591	178.39	0.00587	177.33	0.00591	178.41
15.0	0.01267	355.30	0.01262	353.47	0.01267	355.34
25.0	0.01952	582.73	0.01944	580.05	0.01952	582.77
35.0	0.02198	795.86	0.02187	791.83	0.02198	795.90
45.0	0.02806	993.48	0.02792	988.11	0.02806	993.53
55.0	0.02723	1169.80	0.02708	1162.92	0.02723	1169.91
65.0	0.03132	1327.90	0.03112	1318.60	0.03132	1328.02
75.0	0.02711	1464.70	0.02680	1447.55	0.02711	1464.83
85.0	0.03010	1585.30	0.02959	1559.73	0.03011	1585.55
95.0	0.02671	1679.60	0.02622	1650.40	0.02671	1679.89
105.0	0.02825	1752.40	0.02772	1722.40	0.02826	1752.80
115.0	0.02465	1814.10	0.02439	1795.89	0.02466	1814.66
125.0	0.02561	1867.70	0.02534	1849.19	0.02563	1868.38
135.0	0.02229	1910.20	0.02211	1895.24	0.02230	1910.82
145.0	0.01729	1952.50	0.01709	1930.27	0.01729	1952.63



Figure 4.14.4
Absolute acceleration
response spectra of
layer 1 for damping
ratio of 5%

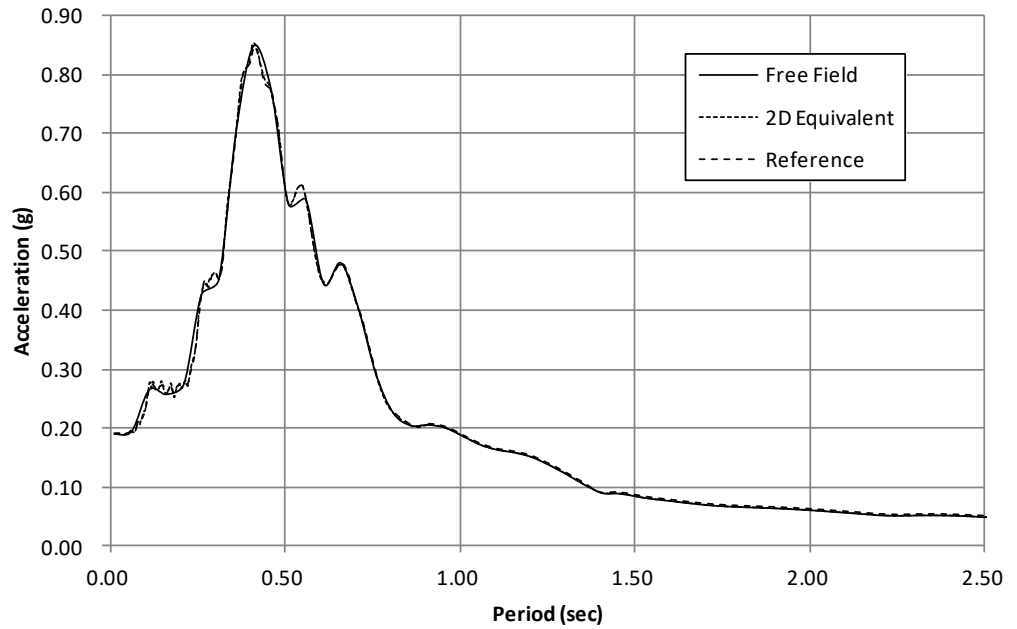
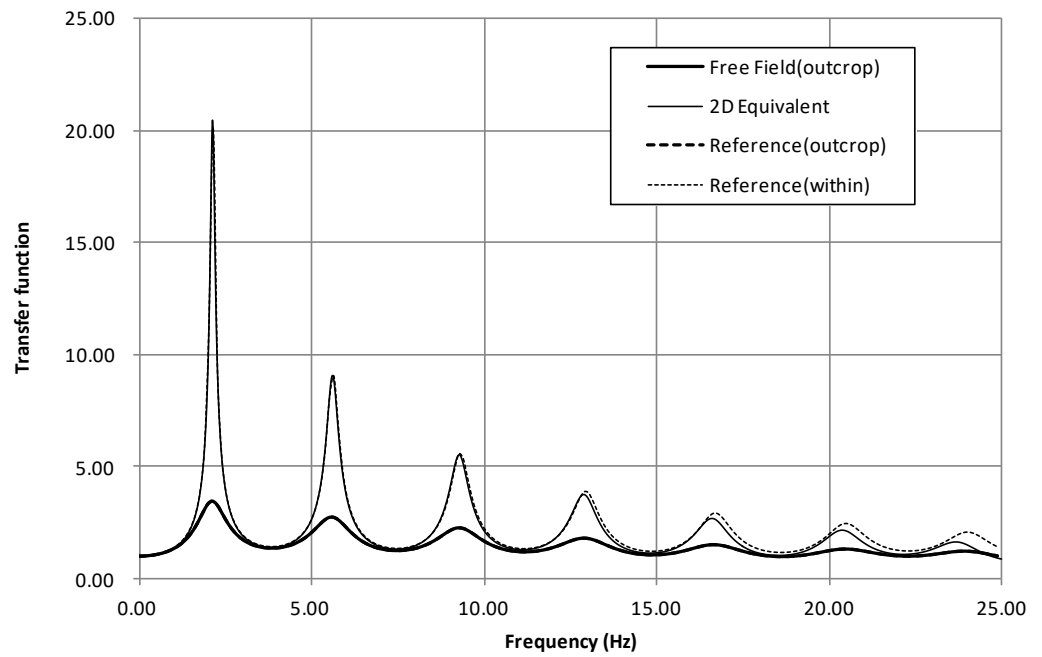


Figure 4.14.5
Transfer function of
acceleration between
layer 1 and layer 17



4.15 Earthquake Response of a Layered Soil Deposit

REFERENCE	Nielsen, A. H. ¹² and Zienkiewicz et al. ¹³
KEYWORDS	Free field element
MODEL FILENAME	Dynamic15.fea

Figure 4.15.1 shows a flat site with horizontal soil layers. The flat site is subjected to enforced acceleration at the bottom. Direct linear transient analyses are carried out for two different modeling cases. In Case 1, the left and right edges of the model are constrained to have identical displacements utilizing multi-point constraints. This gives reference free field response for the infinite flat site. In Case 2, the horizontal soil layers are modeled using free field elements placed at the lateral boundaries. These models should give identical free field responses. Figure 4.15.2 shows the applied acceleration history at the bottom of the model. The same enforced motion is applied for both x and y directions. The acceleration histories at the top surface of both models are compared.

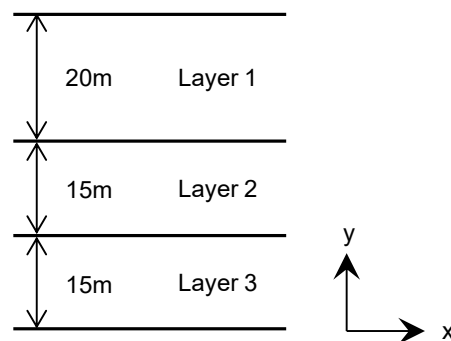


Figure 4.15.1
Flat site profile

Material data 1 (Layer 1)	Young's modulus	$E = 300 \text{ MPa}$
	Poisson's ratio	$\nu = 0.3$
	Unit weight	$\gamma = 19.6 \text{ kN/m}^3$
Material data 2 (Layer 2)	Young's modulus	$E = 700 \text{ MPa}$
	Poisson's ratio	$\nu = 0.4$
	Unit weight	$\gamma = 23.52 \text{ kN/m}^3$
Material data 3 (Layer 3)	Young's modulus	$E = 2000 \text{ MPa}$
	Poisson's ratio	$\nu = 0.25$
	Unit weight	$\gamma = 26.46 \text{ kN/m}^3$
Section	Free field width factor	10^5



Figure 4.15.2
Applied ground
acceleration profile

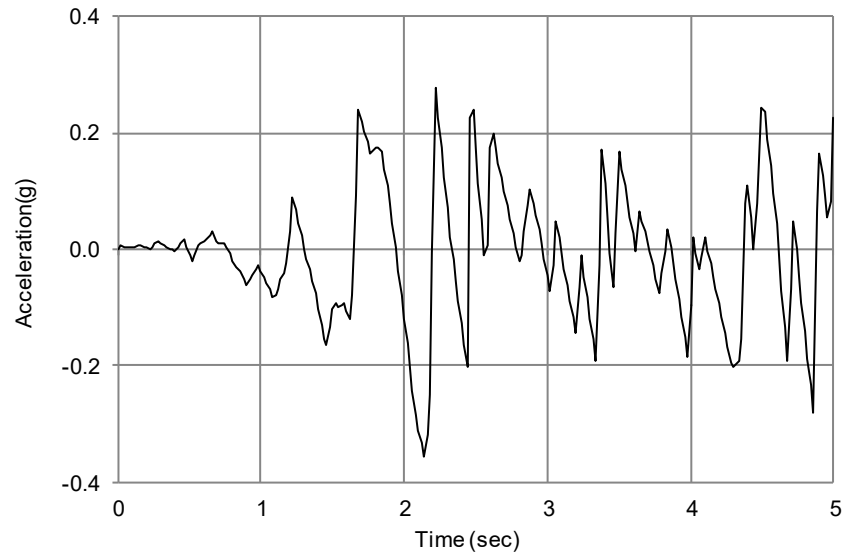


Figure 4.15.3
Horizontal acceleration
history at the top
surface of the model
obtained using free-
field elements and tied
boundary conditions

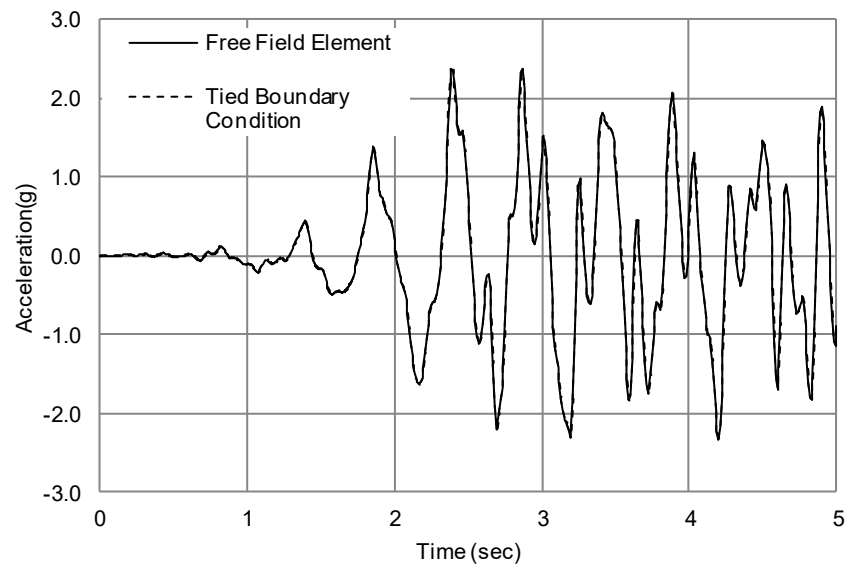
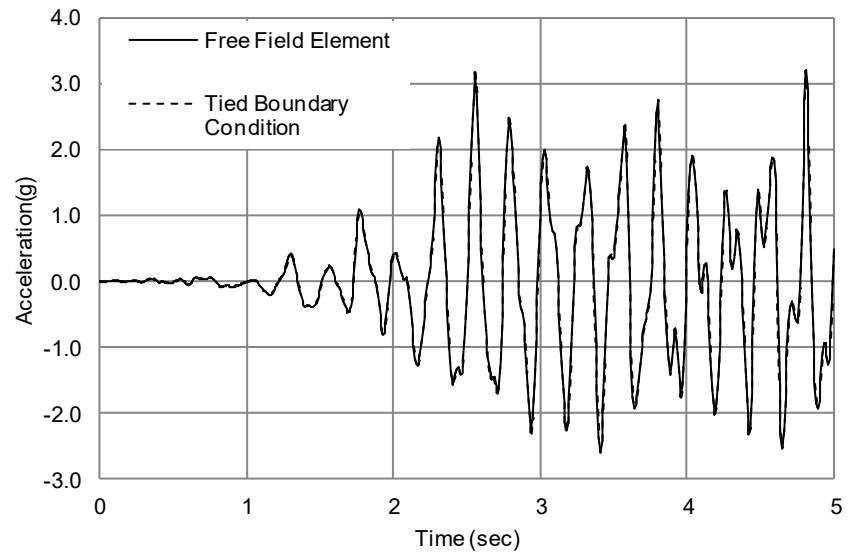




Figure 4.15.4
Vertical acceleration
history at the top
surface of the model
obtained using free-
field elements and tied
boundary conditions





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