



12.1 Uniaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, secondary creep
MODEL FILENAME	MaterialViscoElastic01.fea

Figure 12.1.1 shows a 2D plane stress model undergoing uniaxial tensile stress. Plane stress elements are incorporated to evaluate the secondary creep response.

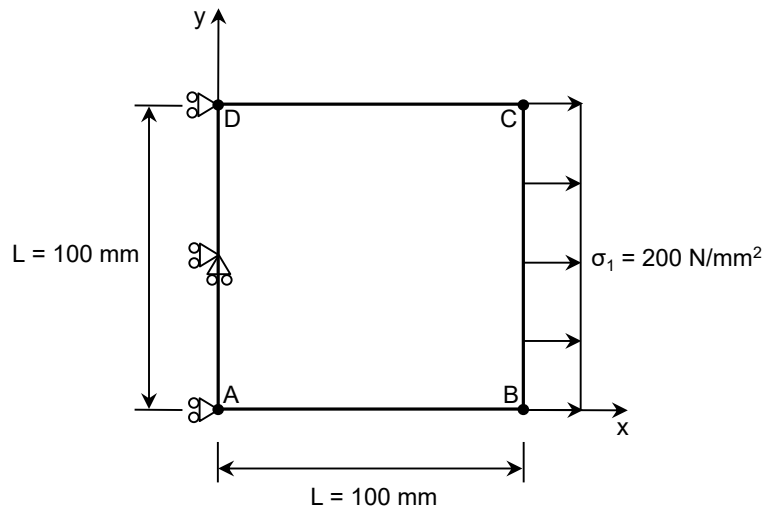


Figure 12.1.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t, A = 3.125 \times 10^{-14}, n = 5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	At midpoint of line AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
Reference creep strain	ε_{xx}	0.01t
	ε_{yy}	-0.005t

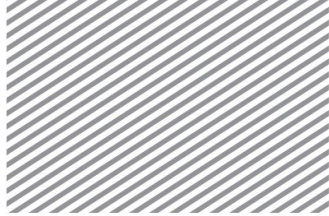


Figure 12.1.2
Comparison of
reference solution and
MaterialViscoElastic01

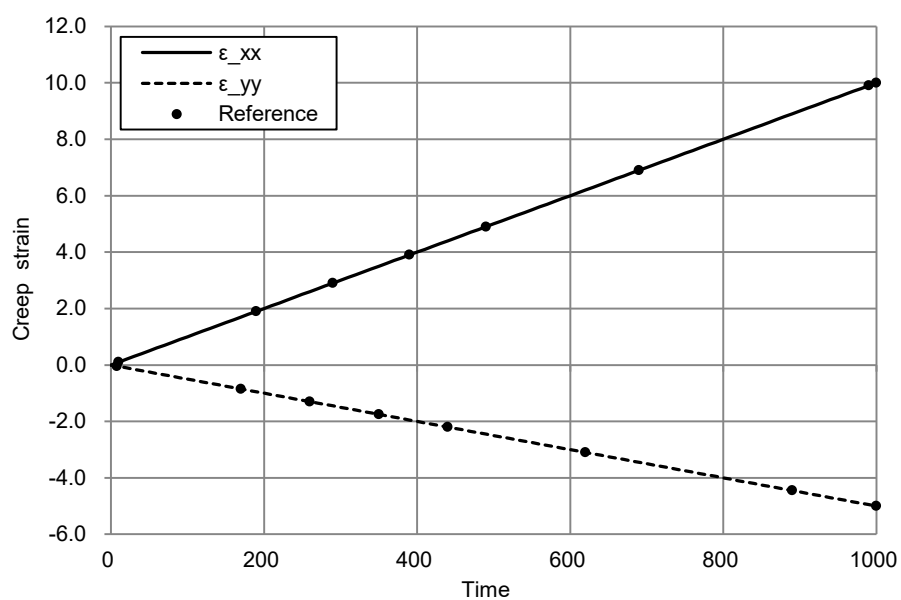


Figure 12.1.3
Result of
MaterialViscoElastic01

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
100	1.00	1.00	-0.50	-0.50
200	2.00	2.00	-1.00	-1.00
300	3.00	3.00	-1.50	-1.50
400	4.00	4.00	-2.00	-2.00
500	5.00	5.00	-2.50	-2.50
600	6.00	6.00	-3.00	-3.00
700	7.00	7.00	-3.50	-3.50
800	8.00	8.00	-4.00	-4.00
900	9.00	9.00	-4.50	-4.50
1000	10.00	10.00	-5.00	-5.00

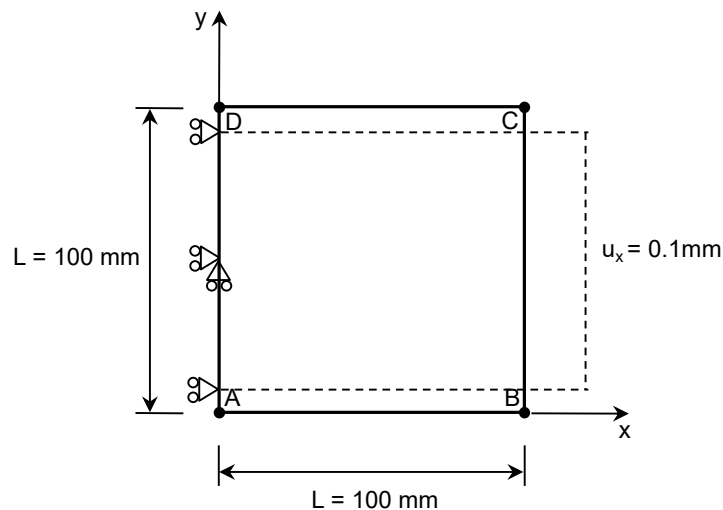


12.2 Uniaxial displacement

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, secondary creep
MODEL FILENAME	MaterialViscoElastic02.fea

Figure 12.2.1 shows a 2D plane stress model undergoing constant uniaxial displacement. Plane stress elements are incorporated to evaluate the secondary creep response.

Figure 12.2.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t, A = 3.125 \times 10^{-14}, n = 5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	At midpoint of line AD	$U_y = 0$
	On line BC	$U_x = 1$
Reference creep stress	σ_{xx}	$\left[(6.25 \times 10^{-10}) + (2.5 \times 10^{-8}) t \right]^{-\frac{1}{4}} \text{ (N/mm}^2\text{)}$



Figure 12.2.2
Comparison of
reference solution and
MaterialViscoElastic02

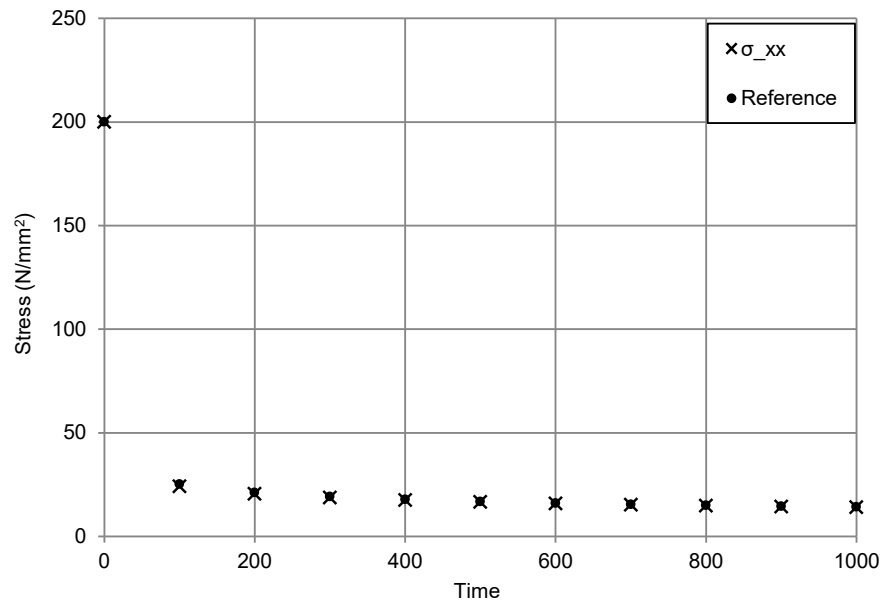


Figure 12.2.3
Result of
MaterialViscoElastic02

Time	σ_{xx}	Reference
100	24.560	25.147
200	20.884	21.147
300	18.946	19.108
400	17.667	17.783
500	16.729	16.818
600	15.997	16.068
700	15.402	15.461
800	14.903	14.953
900	14.476	14.519
1000	14.103	14.142



13.3 Biaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, secondary creep
MODEL FILENAME	MaterialViscoElastic03.fea

Figure 12.3.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the secondary creep response.

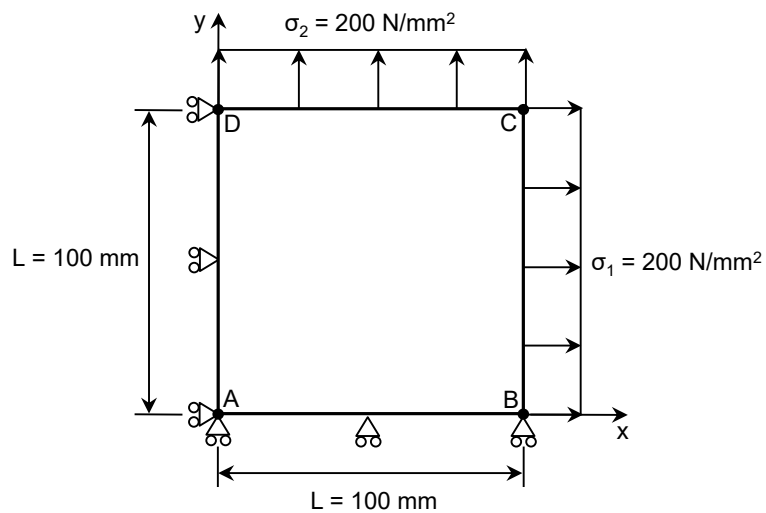


Figure 12.3.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\epsilon = A\sigma^n t, A = 3.125 \times 10^{-14}, n = 5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = 200 \text{ N/mm}^2$
Reference creep strain	ϵ_{xx}	$0.005t$
	ϵ_{yy}	$0.005t$



Figure 12.3.2
Comparison of
reference solution and
MaterialViscoElastic03

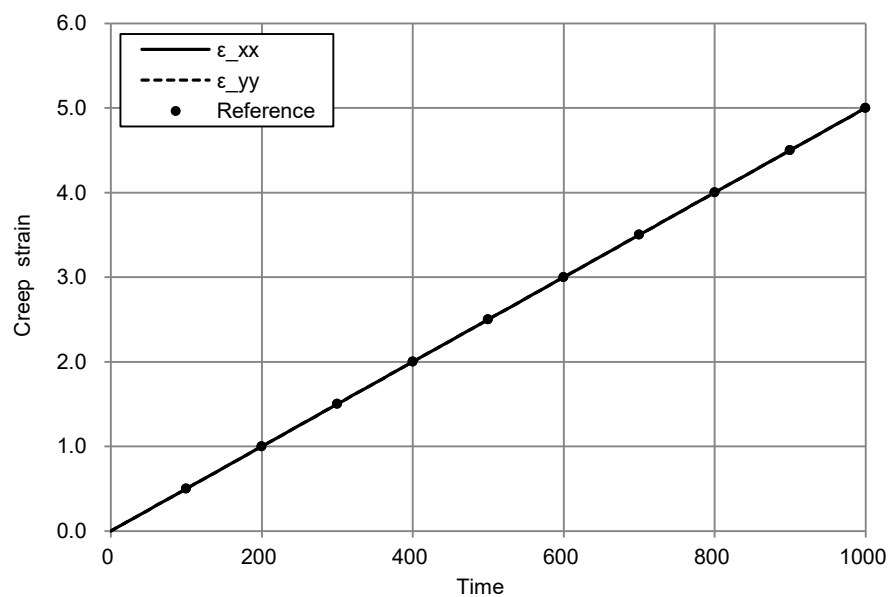
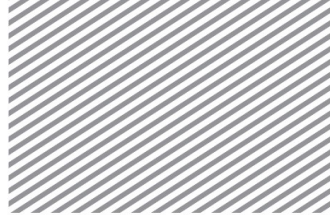


Figure 12.3.3
Result of
MaterialViscoElastic03

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
100	0.50	0.50	0.50	0.50
200	1.00	1.00	1.00	1.00
300	1.50	1.50	1.50	1.50
400	2.00	2.00	2.00	2.00
500	2.50	2.50	2.50	2.50
600	3.00	3.00	3.00	3.00
700	3.50	3.50	3.50	3.50
800	4.00	4.00	4.00	4.00
900	4.50	4.50	4.50	4.50
1000	5.00	5.00	5.00	5.00



12.4 Negative biaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, secondary creep
MODEL FILENAME	MaterialViscoElastic04.fea

Figure 12.4.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the secondary creep response.

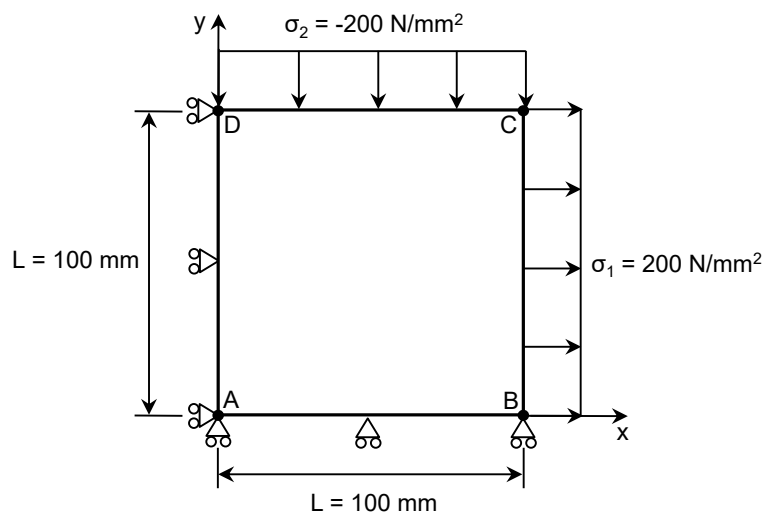


Figure 12.4.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t$, $A = 3.125 \times 10^{-14}$, $n = 5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2$



Reference creep strain	ε_{xx}	$0.135t$
	ε_{yy}	$-0.135t$

Figure 12.4.2
Comparison of
reference solution and
MaterialViscoElastic04

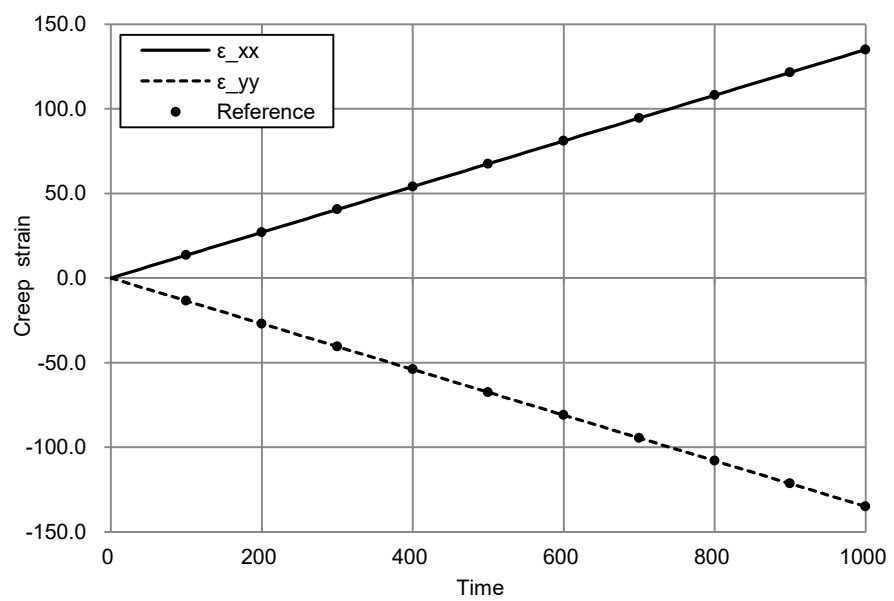
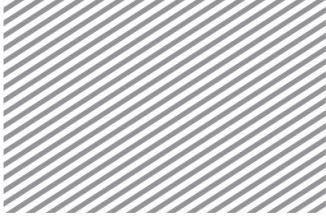
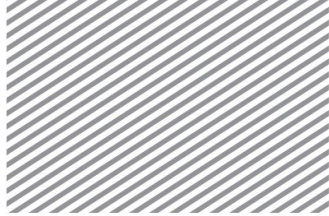


Figure 12.4.3
Result of
MaterialViscoElastic04

Time	ε_{xx}	Reference	ε_{yy}	Reference
100	12.50	13.50	-13.50	-13.50
200	27.00	27.00	-27.00	-27.00
300	40.50	40.50	-40.50	-40.50
400	54.00	54.00	-54.00	-54.00
500	67.50	67.50	-67.50	-67.50
600	81.00	81.00	-81.00	-81.00
700	94.50	94.50	-94.50	-94.50
800	108.00	108.00	-108.00	-108.00



900	121.50	121.50	-121.50	-121.50
1000	135.00	135.00	-135.00	-135.00

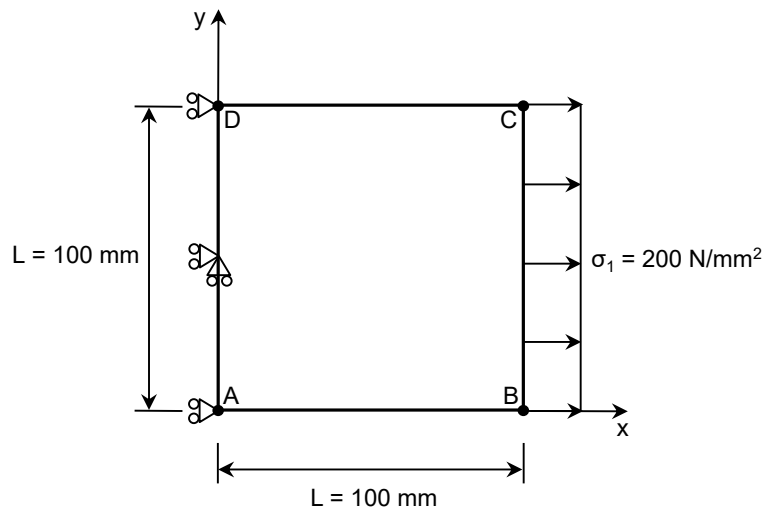


12.5 Uniaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic05.fea

Figure 12.5.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

Figure 12.5.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n=5$, $m=5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	At midpoint of AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
Reference creep strain	ε_{xx}	$0.01\sqrt{t}$
	ε_{yy}	$-0.005\sqrt{t}$



Figure 12.5.2
Comparison of
reference solution and
MaterialViscoElastic05

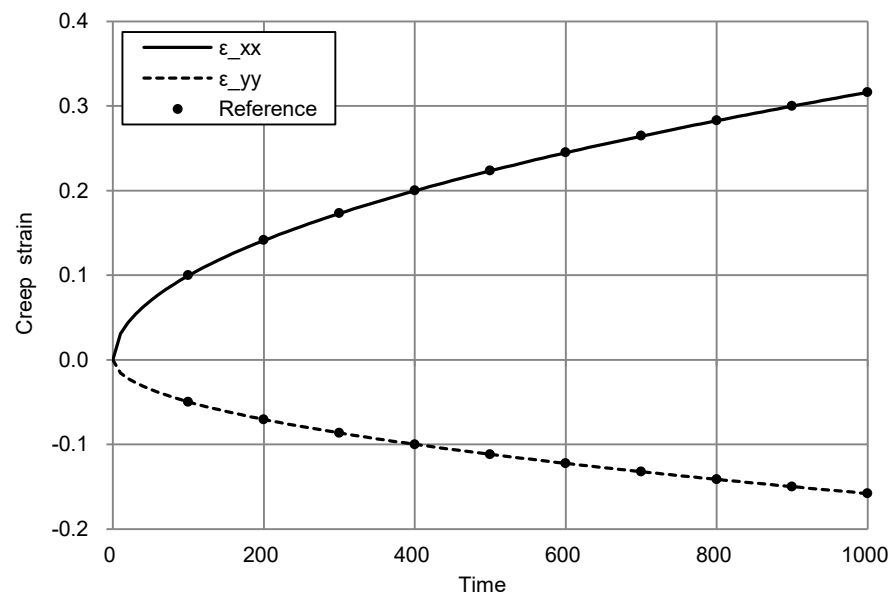
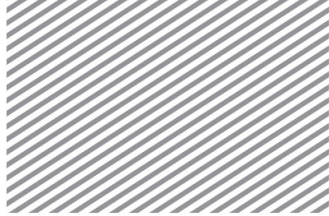


Figure 12.5.3
Result of
MaterialViscoElastic05

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
100	0.099	0.100	-0.050	-0.050
200	0.141	0.141	-0.070	-0.071
300	0.173	0.173	-0.086	-0.087
400	0.200	0.200	-0.100	-0.100
500	0.223	0.224	-0.112	-0.112
600	0.245	0.245	-0.122	-0.122
700	0.264	0.265	-0.132	-0.132
800	0.283	0.283	-0.141	-0.141
900	0.300	0.300	-0.150	-0.150
1000	0.316	0.316	-0.158	-0.158

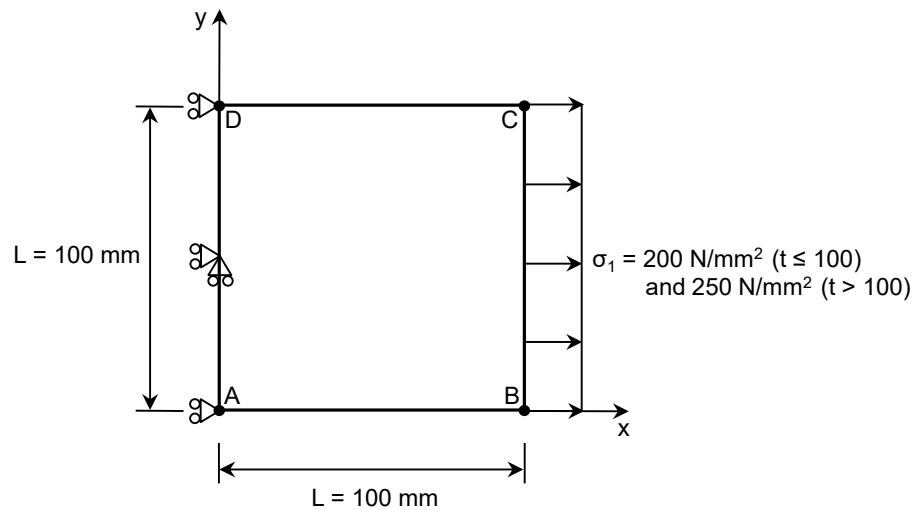


12.6 Uniaxial stepped load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic06.fea

Figure 12.6.1 shows a 2D plane stress model undergoing stepped tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

Figure 12.6.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n = 5$, $m = 5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	At midpoint of AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$ ($t \leq 100$), 250 N/mm^2 ($t > 100$)
Reference creep strain	ε_{xx}	$0.01\sqrt{t}$ ($t \leq 100$), $0.03052\sqrt{t - 89.262}$ ($t > 100$)



Figure 12.6.2
Comparison of
reference solution and
MaterialViscoElastic06

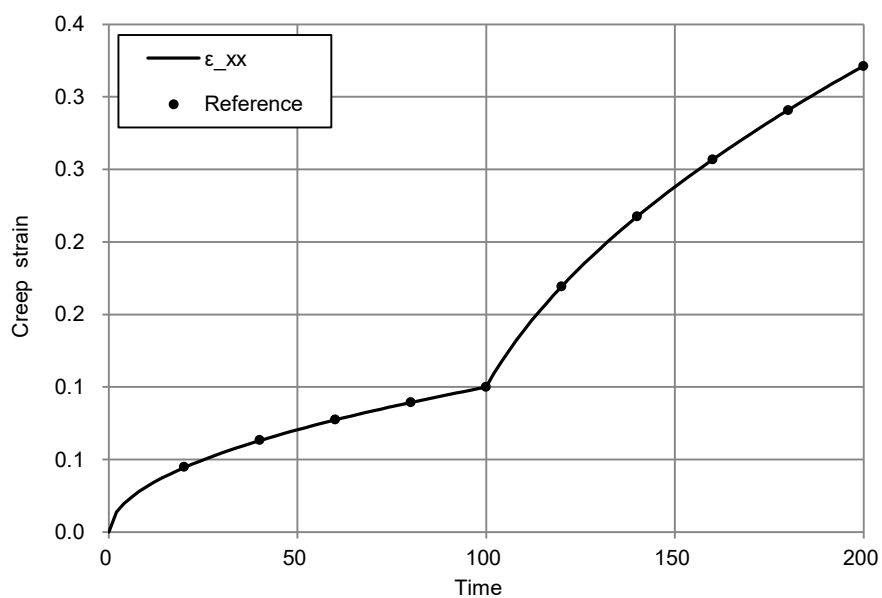


Figure 12.6.3
Result of
MaterialViscoElastic06

Time	ϵ_{xx}	Reference
20	0.044	0.045
40	0.063	0.063
60	0.077	0.077
80	0.089	0.089
100	0.100	0.100
120	0.169	0.169
140	0.218	0.217
160	0.257	0.257
180	0.291	0.291
200	0.321	0.321

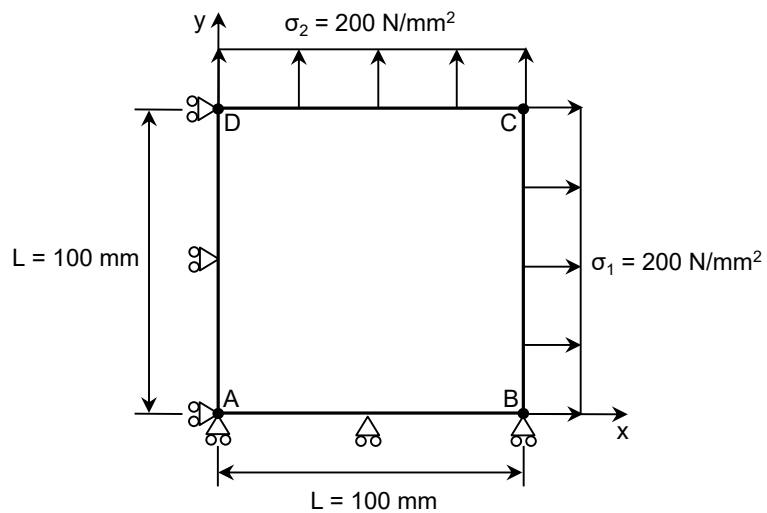


12.7 Biaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic07.fea

Figure 12.7.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

Figure 12.7.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n = 5$, $m = 5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = 200 \text{ N/mm}^2$
Reference creep strain	ε_{xx}	$0.005\sqrt{t}$
	ε_{yy}	$0.005\sqrt{t}$

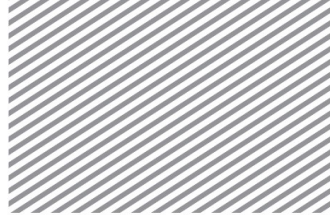


Figure 12.7.2
Comparison of
reference solution and
MaterialViscoElastic07

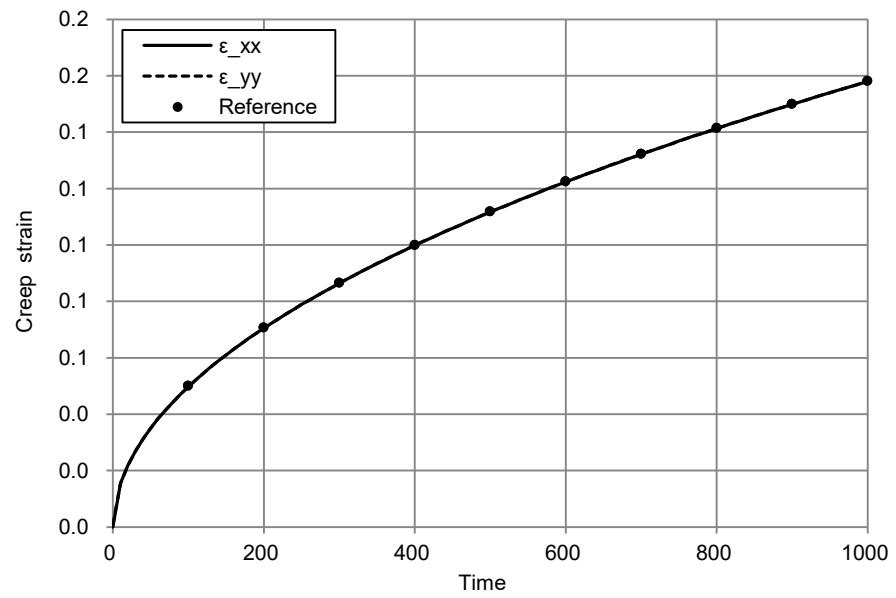


Figure 12.7.3
Result of
MaterialViscoElastic07

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
100	0.050	0.050	-0.050	0.050
200	0.070	0.071	-0.070	0.071
300	0.086	0.087	-0.086	0.087
400	0.100	0.100	-0.100	0.100
500	0.112	0.112	-0.112	0.112
600	0.122	0.122	-0.122	0.122
700	0.132	0.132	-0.132	0.132
800	0.141	0.141	-0.141	0.141
900	0.150	0.150	-0.150	0.150
1000	0.158	0.158	-0.158	0.158

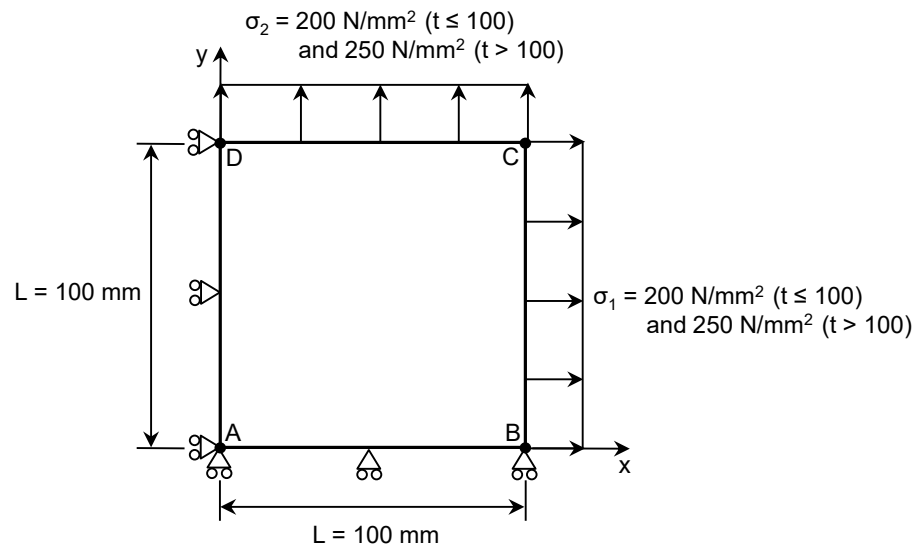


12.8 Biaxial stepped load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic08.fea

Figure 12.8.1 shows a 2D plane stress model undergoing biaxial stepped tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

Figure 12.8.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n=5$, $m=5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$ ($t \leq 100$), 250 N/mm^2 ($t > 100$)
	On line CD	$\sigma_2 = 200 \text{ N/mm}^2$ ($t \leq 100$), 250 N/mm^2 ($t > 100$)
Reference creep strain	ε_{xx}	$0.005\sqrt{t}$ ($t \leq 100$),
		$0.01529\sqrt{t-89.262}$ ($t > 100$)



Figure 12.8.2
Comparison of
reference solution and
MaterialViscoElastic08

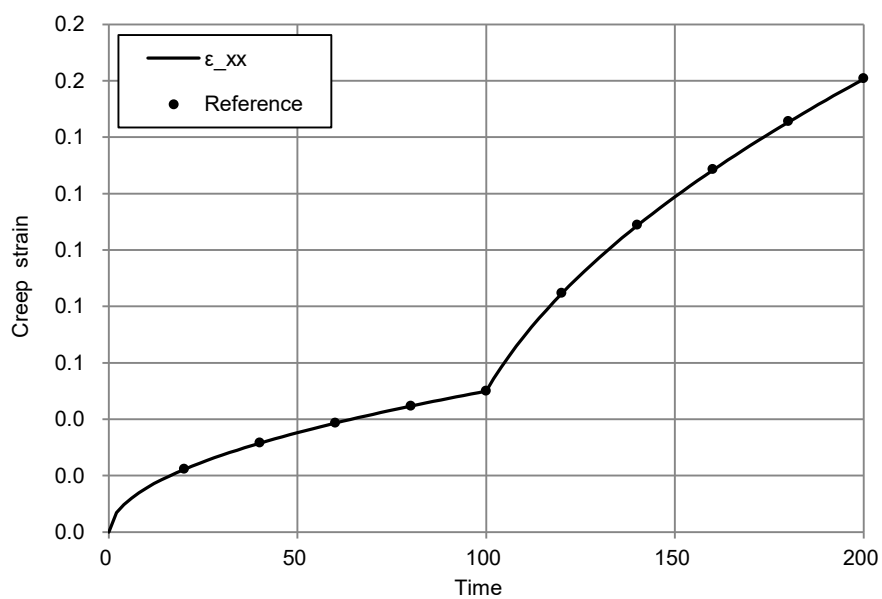
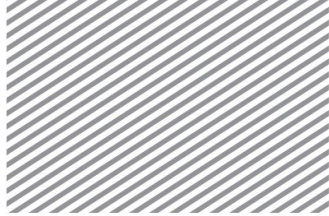


Figure 12.8.3
Result of
MaterialViscoElastic08

Time	ϵ_{xx}	Reference
20	0.022	0.022
40	0.032	0.032
60	0.039	0.039
80	0.045	0.045
100	0.050	0.050
120	0.085	0.085
140	0.109	0.109
160	0.128	0.129
180	0.145	0.146
200	0.161	0.161



12.9 Negative biaxial load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic09.fea

Figure 12.9.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

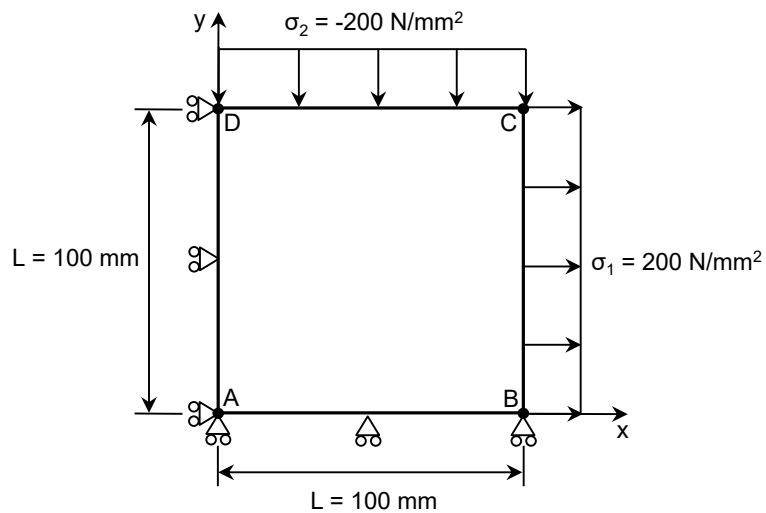


Figure 12.9.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n=5$, $m=5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2$
Reference creep strain	ε_{xx}	$0.135\sqrt{t}$



Figure 12.9.2
Comparison of
reference solution and
MaterialViscoElastic09

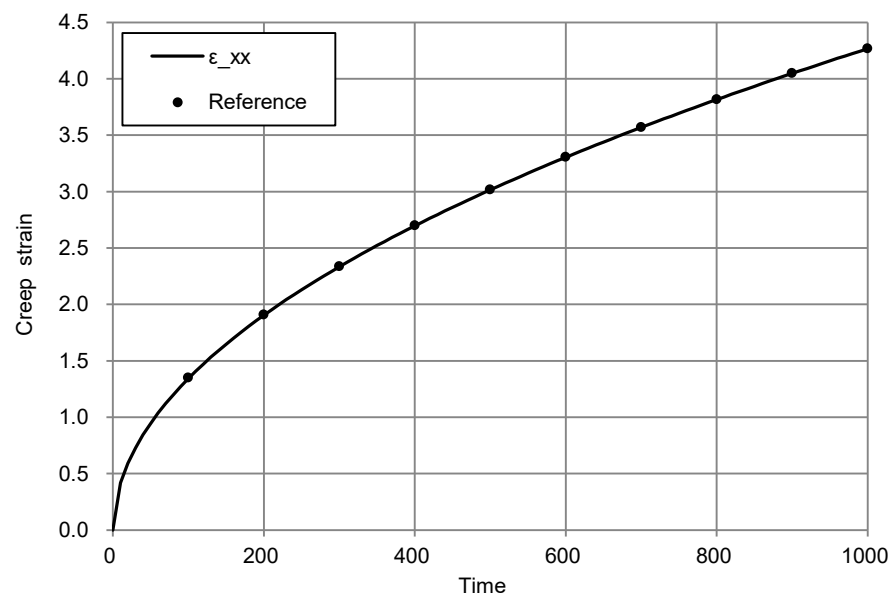
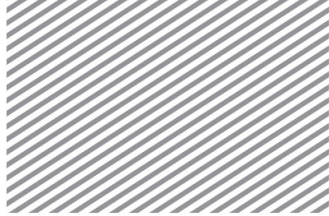


Figure 12.9.3
Result of
MaterialViscoElastic09

Time	ϵ_{xx}	Reference
100	1.342	1.350
200	1.904	1.909
300	2.334	2.338
400	2.696	2.700
500	3.015	3.019
600	3.304	3.307
700	3.569	3.572
800	3.816	3.818
900	4.048	4.050
1000	4.267	4.269

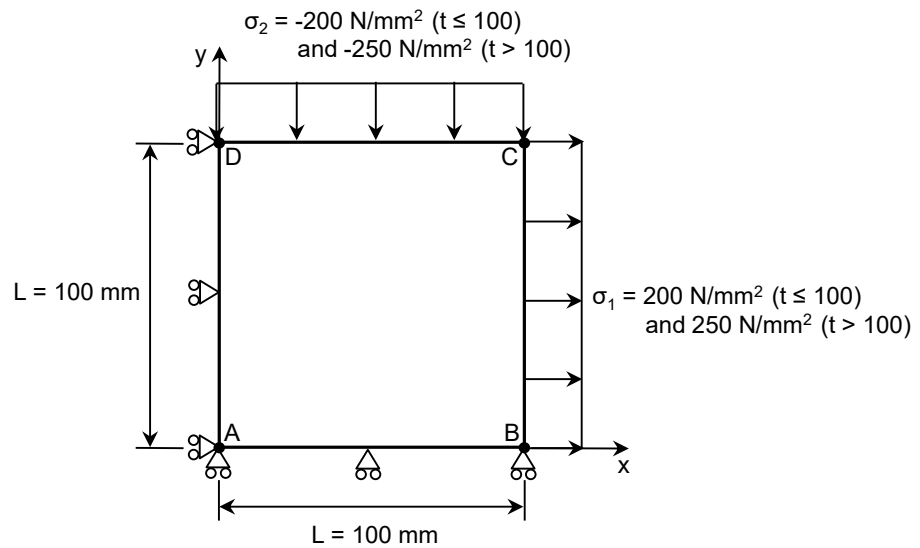


12.10 Negative biaxial stepped load

REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary creep
MODEL FILENAME	MaterialViscoElastic10.fea

Figure 12.10.1 shows a 2D plane stress model undergoing biaxial stepped tensile stress. Plane stress elements are incorporated to evaluate the primary creep response.

Figure 12.10.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t^m$, $A = 3.125 \times 10^{-14}$, $n = 5$, $m = 5$ (N/mm^2)
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$ ($t \leq 100$), 250 N/mm^2 ($t > 100$)
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2$ ($t \leq 100$), -250 N/mm^2 ($t > 100$)

**Reference creep strain** ϵ_{xx}

$$0.135\sqrt{t} \quad (t \leq 100),$$
$$0.4120\sqrt{t - 89.2605} \quad (t > 100)$$

Figure 12.10.2
Comparison of
reference solution and
MaterialViscoElastic10

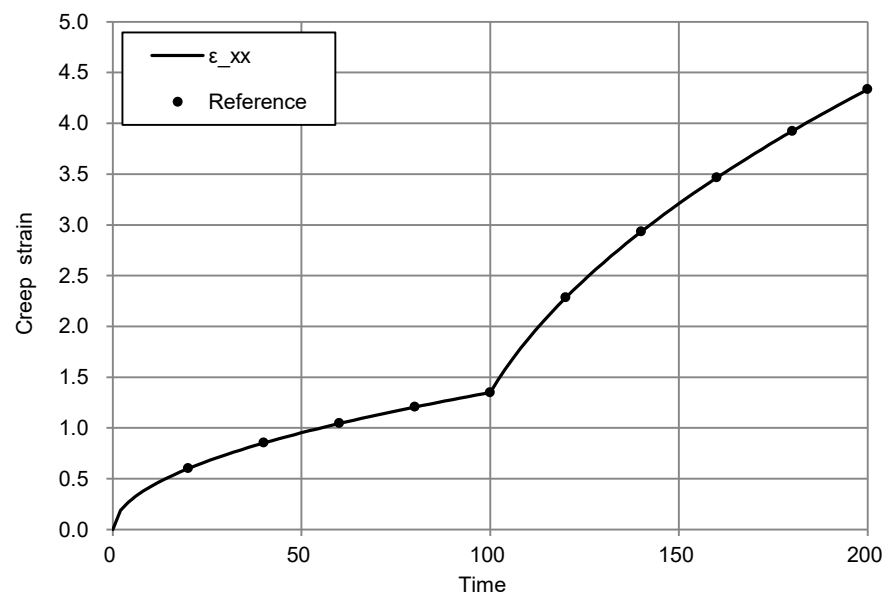


Figure 12.10.3
Result of
MaterialViscoElastic10

Time	ϵ_{xx}	Reference
20	0.600	0.604
40	0.851	0.854
60	1.044	1.046
80	1.206	1.207
100	1.348	1.350
120	2.282	2.284
140	2.933	2.935
160	3.464	3.465
180	3.923	3.925
200	4.334	4.336

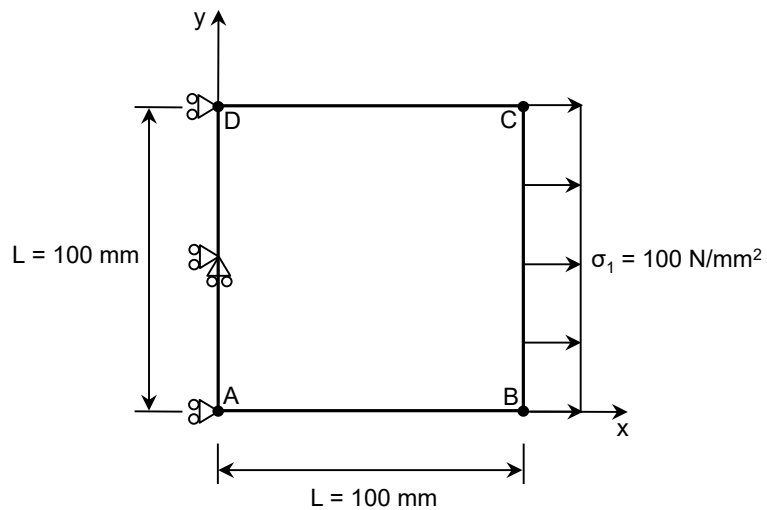


12.11 Uniaxial load

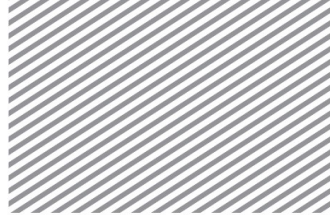
REFERENCE	NAFEM [13-1]
ELEMENT	Plane stress element, Maxwell-kelvin element, primary-secondary creep
MODEL FILENAME	MaterialViscoElastic11.fea

Figure 12.11.1 shows a 2D plane stress model undergoing uniaxial tensile stress. Plane stress elements are incorporated to evaluate the primary and secondary creep response.

Figure 12.11.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^3 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law,	$\varepsilon = A_1 \sigma^n t + A_2 \sigma^n t^m$
	Primary-secondary	$A_1 = 10^{-16}, A_2 = 10^{-14}, n=5, m=0.5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	At midpoint of AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 100 \text{ N/mm}^2$



Reference creep strain	ε_{xx}	$0.0001(0.01t + \sqrt{t})$
	ε_{yy}	$-0.5 \varepsilon_{xx}$

Figure 12.11.2
Comparison of
reference solution and
MaterialViscoElastic11

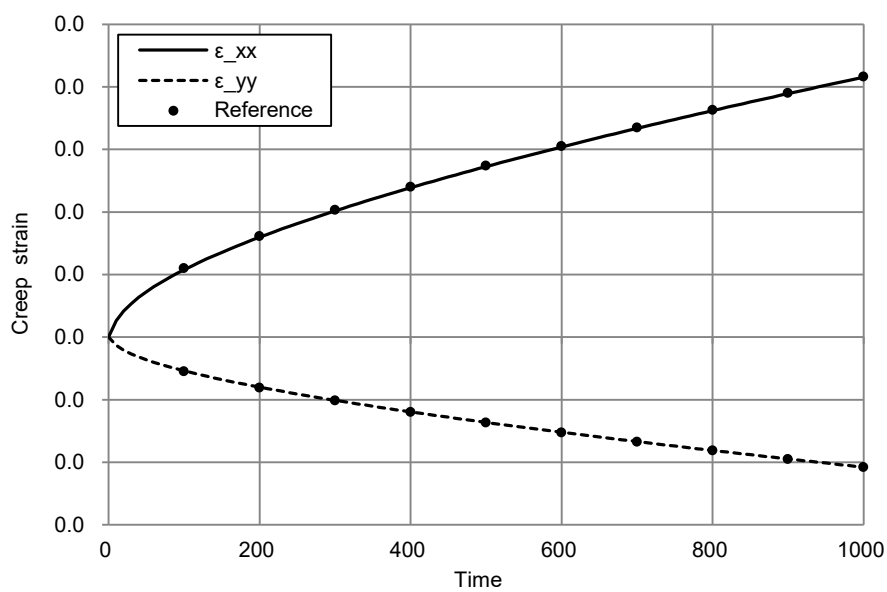
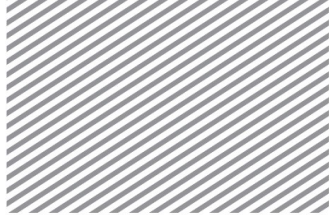


Figure 12.7.3
Result of
MaterialViscoElastic11

Time	ε_{xx}	Reference	ε_{yy}	Reference
100	0.0011	0.0011	-0.0005	-0.0006
200	0.0016	0.0016	-0.0008	-0.0008
300	0.0020	0.0020	-0.0010	-0.0010
400	0.0024	0.0024	-0.0012	-0.0012
500	0.0027	0.0027	-0.0014	-0.0014
600	0.0030	0.0030	-0.0015	-0.0015
700	0.0033	0.0033	-0.0017	-0.0017
800	0.0036	0.0036	-0.0018	-0.0018
900	0.0039	0.0039	-0.0019	-0.0020
1000	0.0042	0.0042	-0.0021	-0.0021

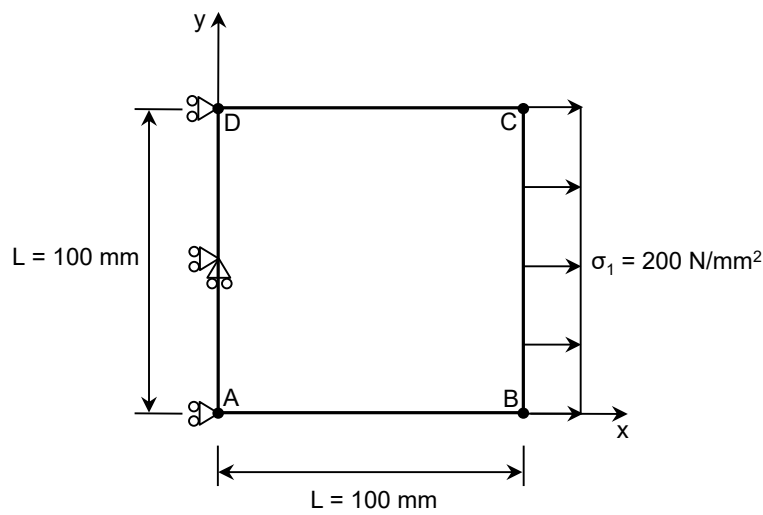


12.12 Uniaxial load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic12.fea

Figure 12.12.1 shows a 2D plane stress model undergoing uniaxial tensile stress. Plane stress elements are incorporated to evaluate the creep response.

Figure 7.12.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_o) = 1 - e^{-(t-t_o)/50}$
Boundary condition	On line AD	$U_x = 0$
	At midpoint of AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
Reference creep strain	ϵ_{xx}	$0.01\phi(t, t_o)$
	ϵ_{yy}	$-0.003\phi(t, t_o)$



Figure 12.12.1
Comparison of
reference solution and
MaterialViscoElastic12

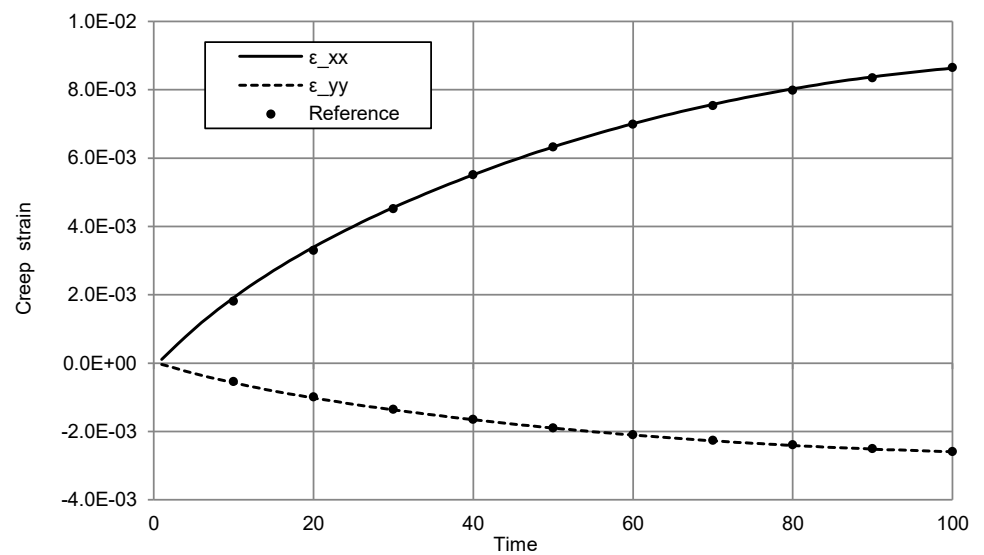
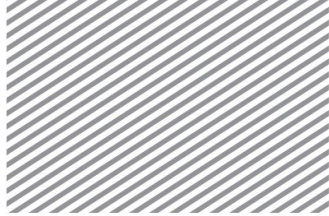


Figure 12.12.1
Result of
MaterialViscoElastic12

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0019	0.0018	-0.0006	-0.0005
20	0.0034	0.0033	-0.0010	-0.0010
30	0.0046	0.0045	-0.0014	-0.0014
40	0.0055	0.0055	-0.0017	-0.0017
50	0.0063	0.0063	-0.0019	-0.0019
60	0.0070	0.0070	-0.0021	-0.0021
70	0.0076	0.0075	-0.0023	-0.0023
80	0.0080	0.0080	-0.0024	-0.0024
90	0.0084	0.0083	-0.0025	-0.0025
100	0.0086	0.0086	-0.0026	-0.0026

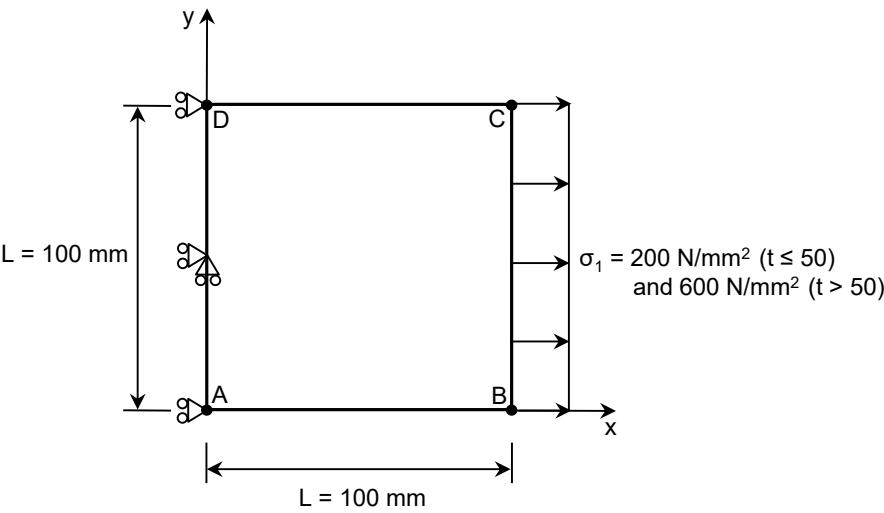


12.13 Stepped load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic13.fea

Figure 12.13.1 shows a 2D plane stress model undergoing stepped tensile stress. Plane stress elements are incorporated to evaluate the creep response.

Figure 12.13.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_o) = 1 - e^{-(t-t_o)/50}$
Boundary condition	On line AD	$U_x = 0$
	At midpoint of AD	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2 (t \leq 50) , 600 \text{ N/mm}^2 (t > 50)$
Reference creep strain	ϵ_{xx}	$0.01\phi(t, 0) + 0.02\phi(t, 50)$
	ϵ_{yy}	$-0.003\phi(t, 0) - 0.006\phi(t, 50)$



Figure 12.13.1
Comparison of
reference solution and
MaterialViscoElastic13

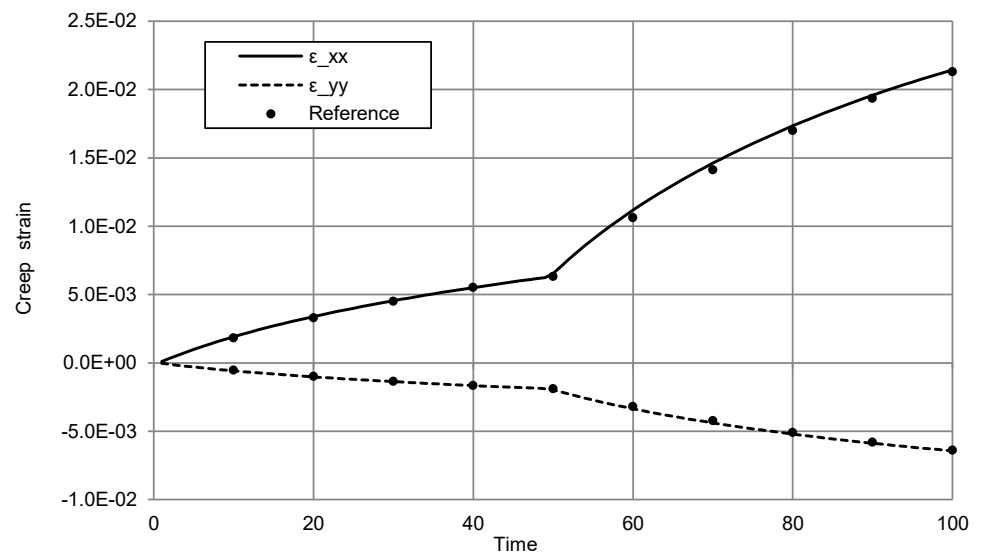
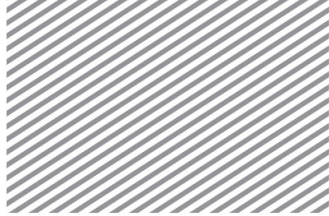


Figure 12.13.1
Result of
MaterialViscoElastic13

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0019	0.0018	-0.0006	-0.0005
20	0.0034	0.0033	-0.0010	-0.0010
30	0.0046	0.0045	-0.0014	-0.0014
40	0.0055	0.0055	-0.0017	-0.0017
50	0.0065	0.0063	-0.0020	-0.0019
60	0.0112	0.0106	-0.0034	-0.0032
70	0.0146	0.0141	-0.0044	-0.0042
80	0.0173	0.0170	-0.0052	-0.0051
90	0.0196	0.0194	-0.0059	-0.0058
100	0.0214	0.0213	-0.0064	-0.0064



12.14 Biaxial load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic14.fea

Figure 12.14.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the creep response.

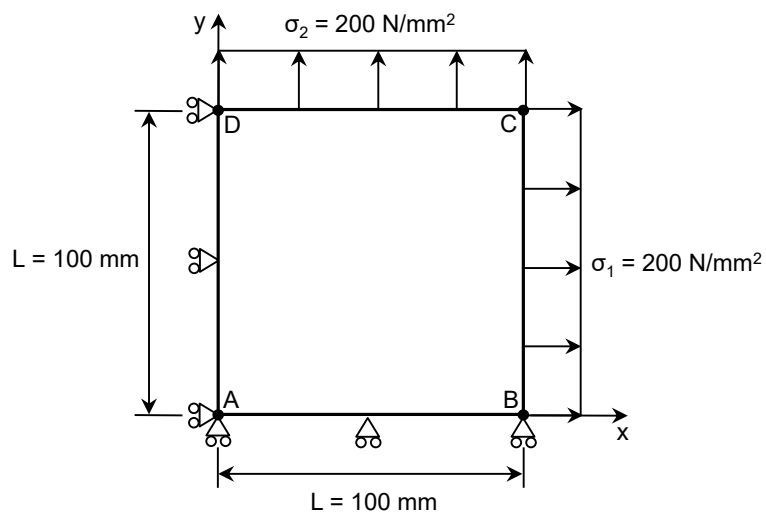


Figure 12.14.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_0) = 1 - e^{-(t-t_0)/50}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = 200 \text{ N/mm}^2$
Reference creep strain	ϵ_{xx}	$0.007\phi(t, 0)$
	ϵ_{yy}	$0.007\phi(t, 0)$



Figure 12.14.1
Comparison of
reference solution and
MaterialViscoElastic14

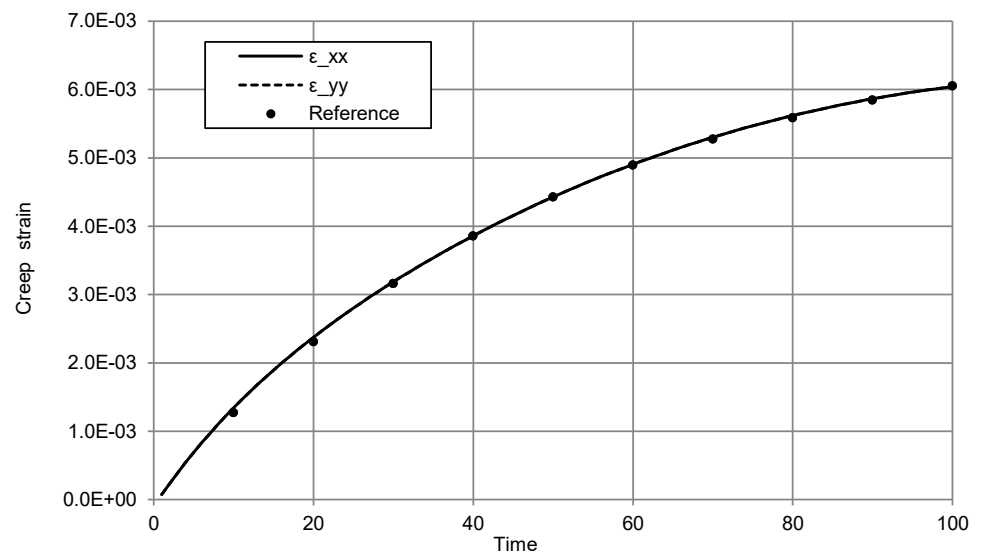


Figure 12.14.1
Result of
MaterialViscoElastic14

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0013	0.0013	0.0013	0.0013
20	0.0022	0.0023	0.0022	0.0023
30	0.0032	0.0032	0.0032	0.0032
40	0.0037	0.0039	0.0037	0.0039
50	0.0044	0.0044	0.0044	0.0044
60	0.0049	0.0049	0.0049	0.0049
70	0.0053	0.0053	0.0053	0.0053
80	0.0056	0.0056	0.0056	0.0056
90	0.0059	0.0058	0.0059	0.0058
100	0.0060	0.0061	0.0060	0.0061

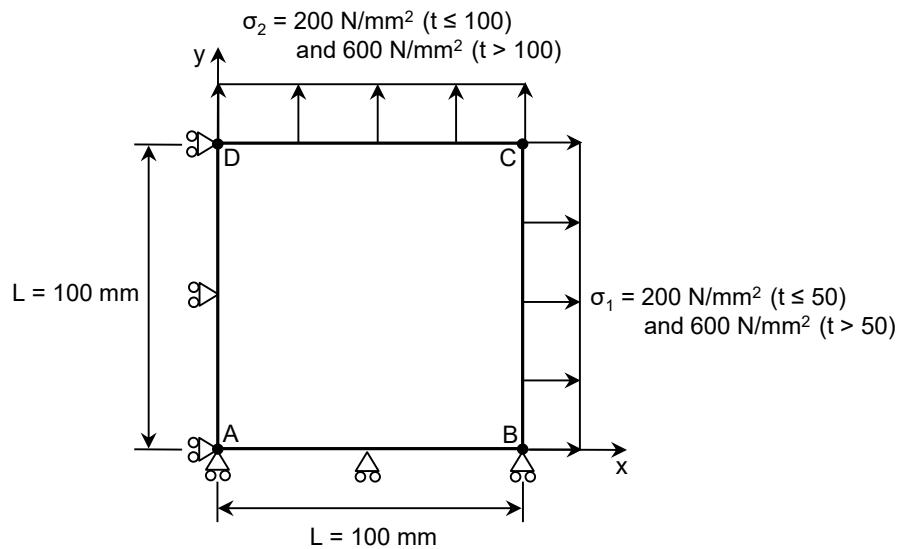


12.15 Biaxial stepped load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic15.fea

Figure 12.15.1 shows a 2D plane stress model undergoing biaxial stepped tensile stress. Plane stress elements are incorporated to evaluate the creep response.

Figure 12.15.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_0) = 1 - e^{-(t-t_0)/50}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2 (t \leq 50), 600 \text{ N/mm}^2 (t > 50)$
	On line CD	$\sigma_2 = 200 \text{ N/mm}^2 (t \leq 50), 600 \text{ N/mm}^2 (t > 50)$
Reference creep strain	ϵ_{xx}	$0.007\phi(t, 0) + 0.014\phi(t, 50)$
	ϵ_{yy}	$0.007\phi(t, 0) + 0.014\phi(t, 50)$



Figure 12.15.1
Comparison of
reference solution and
MaterialViscoElastic15

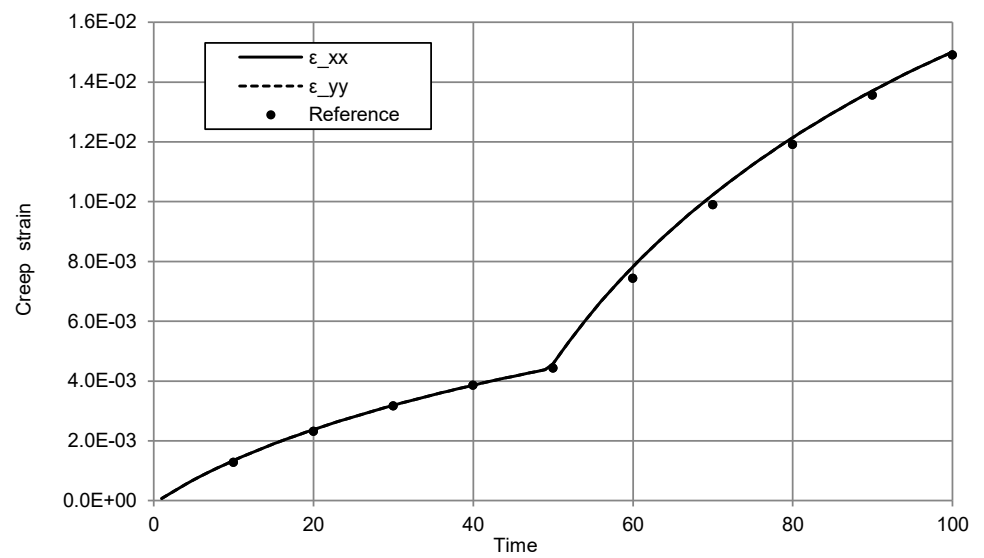
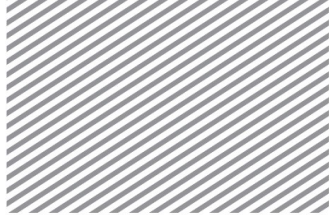


Figure 12.15.1
Result of
MaterialViscoElastic15

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0013	0.0013	0.0013	0.0013
20	0.0024	0.0023	0.0024	0.0023
30	0.0032	0.0032	0.0032	0.0032
40	0.0039	0.0039	0.0039	0.0039
50	0.0046	0.0044	0.0046	0.0044
60	0.0078	0.0074	0.0078	0.0074
70	0.0102	0.0099	0.0102	0.0099
80	0.0121	0.0119	0.0121	0.0119
90	0.0137	0.0136	0.0137	0.0136
100	0.0150	0.0149	0.0150	0.0149



12.16 Negative biaxial load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic16.fea

Figure 12.16.1 shows a 2D plane stress model undergoing biaxial tensile stress. Plane stress elements are incorporated to evaluate the creep response.

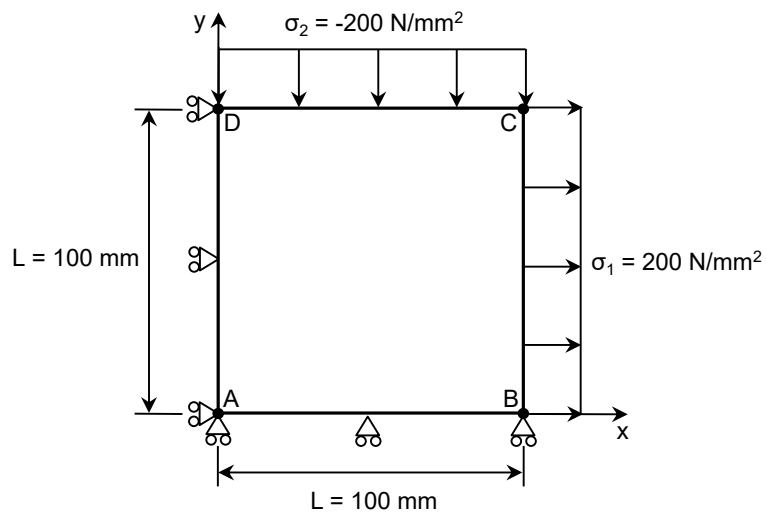


Figure 12.16.1
2D plane stress model

Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep law, secondary	$\varepsilon = A\sigma^n t, A = 3.125 \times 10^{-14}, n = 5 \text{ (N/mm}^2\text{)}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2$
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2$
Reference creep strain	ε_{xx}	$0.013\phi(t, 0)$
	ε_{yy}	$-0.013\phi(t, 0)$



Figure 12.16.2
Comparison of
reference solution and
MaterialViscoElastic16

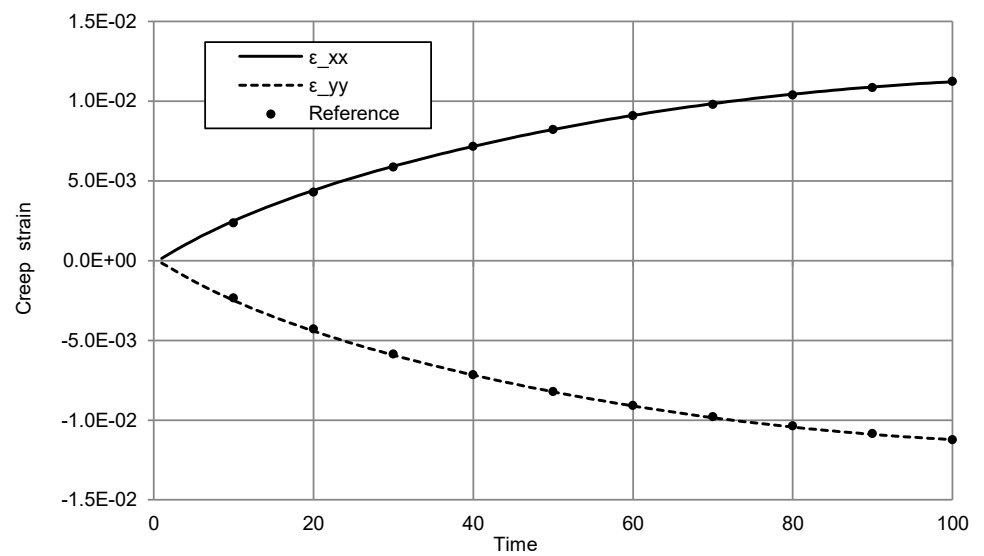
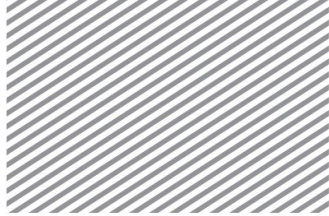


Figure 12.16.3
Result of
MaterialViscoElastic16

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0025	0.0024	-0.0025	-0.0024
20	0.0044	0.0043	-0.0044	-0.0043
30	0.0059	0.0059	-0.0059	-0.0059
40	0.0072	0.0072	-0.0072	-0.0072
50	0.0082	0.0082	-0.0082	-0.0082
60	0.0089	0.0091	-0.0089	-0.0091
70	0.0098	0.0098	-0.0098	-0.0098
80	0.0104	0.0104	-0.0104	-0.0104
90	0.0109	0.0109	-0.0109	-0.0109
100	0.0112	0.0112	-0.0112	-0.0112

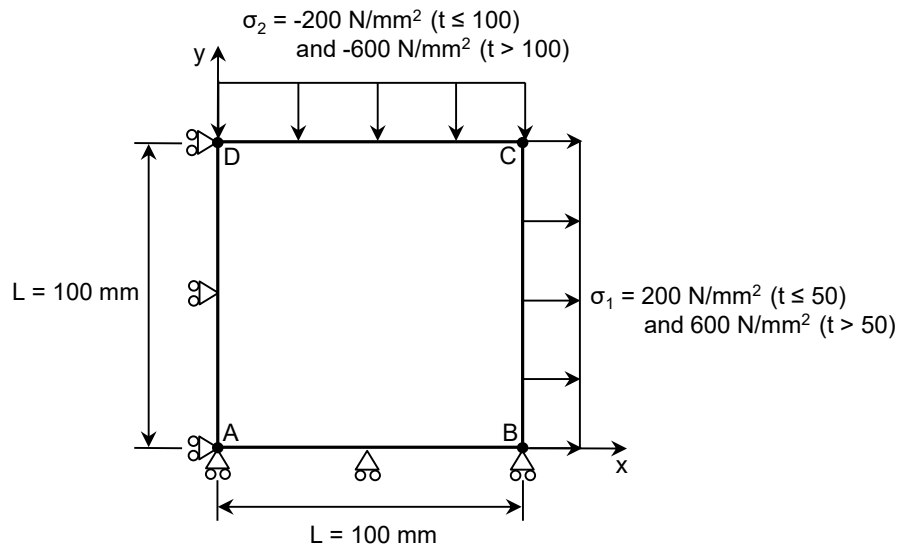


12.17 Negative biaxial stepped load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic17.fea

Figure 12.17.1 shows a 2D plane stress model undergoing biaxial stepped tensile stress. Plane stress elements are incorporated to evaluate the creep response.

Figure 12.17.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_0) = 1 - e^{-(t-t_0)/50}$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2 (t \leq 50), 600 \text{ N/mm}^2 (t > 50)$
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2 (t \leq 50), -600 \text{ N/mm}^2 (t > 50)$
Reference creep strain	ϵ_{xx}	$0.013\phi(t, 0) + 0.026\phi(t, 50)$
	ϵ_{yy}	$-0.013\phi(t, 0) - 0.026\phi(t, 50)$



Figure 12.17.2
Comparison of
reference solution and
MaterialViscoelastic17

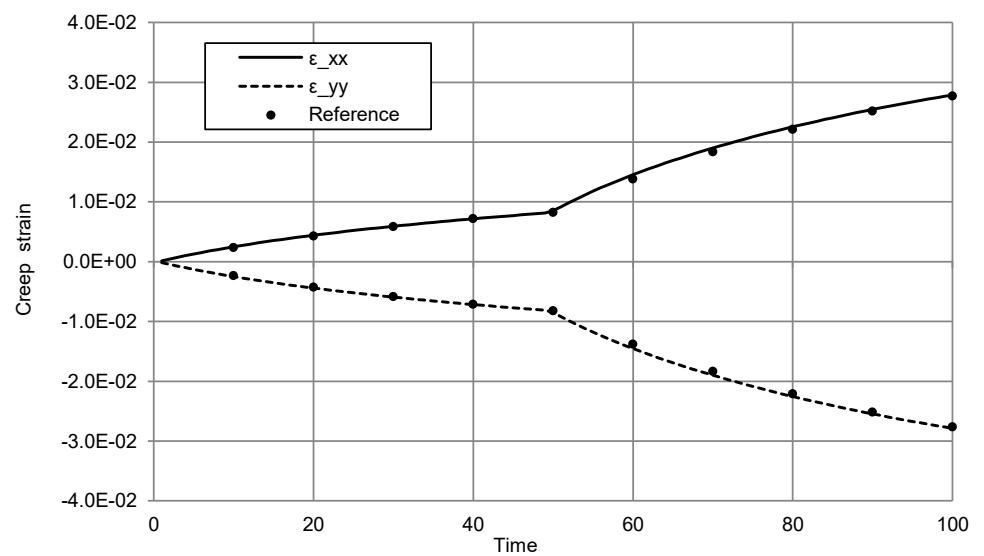
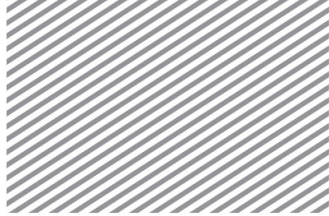


Figure 12.17.3
Result of
MaterialViscoElastic17

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0025	0.0024	-0.0025	-0.0024
20	0.0044	0.0043	-0.0044	-0.0043
30	0.0059	0.0059	-0.0059	-0.0059
40	0.0072	0.0072	-0.0072	-0.0072
50	0.0085	0.0082	-0.0085	-0.0082
60	0.0145	0.0138	-0.0145	-0.0138
70	0.0190	0.0184	-0.0190	-0.0184
80	0.0225	0.0221	-0.0225	-0.0221
90	0.0255	0.0252	-0.0255	-0.0252
100	0.0279	0.0277	-0.0279	-0.0277

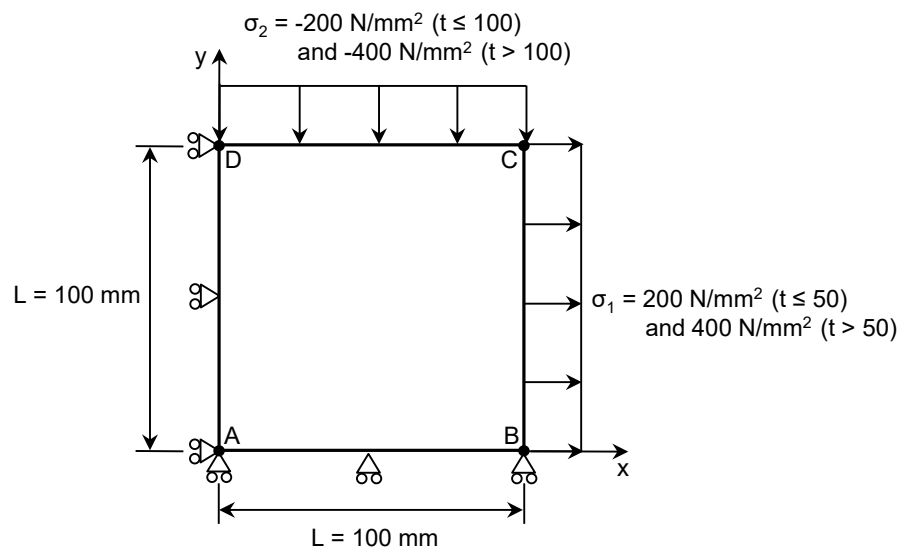


12.18 Time dependent negative biaxial stepped load

REFERENCE	Analytical solution
ELEMENT	Plane stress element, generalized kelvin model
MODEL FILENAME	MaterialViscoElastic18.fea

Figure 12.17.1 shows a 2D plane stress model undergoing biaxial stepped tensile stress. Plane stress elements are incorporated to evaluate the creep, shrinkage and time dependent elastic modulus response.

Figure 12.18.1
2D plane stress model



Material data	Young's modulus	$E = 200.0 \times 10^2 \text{ N/mm}^2 (t \leq 50)$ $100.0 \times 10^2 \text{ N/mm}^2 (t > 50)$
	Poisson's ratio	$\nu = 0.3$
	Creep coefficient	$\phi(t, t_o) = 1 - e^{-(t-t_o)/50}$
	Shrinkage	$\varepsilon_{sh}(t) = -0.05(1 - e^{-t/50})$
Boundary condition	On line AD	$U_x = 0$
	On line AB	$U_y = 0$
Loading	On line BC	$\sigma_1 = 200 \text{ N/mm}^2 (t \leq 50), 400 \text{ N/mm}^2 (t > 50)$
	On line CD	$\sigma_2 = -200 \text{ N/mm}^2 (t \leq 50), -400 \text{ N/mm}^2 (t > 50)$



Reference creep strain	ϵ_{xx}	$0.013\phi(t,0)+0.026\phi(t,50)$
	ϵ_{yy}	$-0.013\phi(t,0)-0.026\phi(t,50)$

Figure 12.18.2
Comparison of
reference creep
solution and
MaterialViscoelastic18

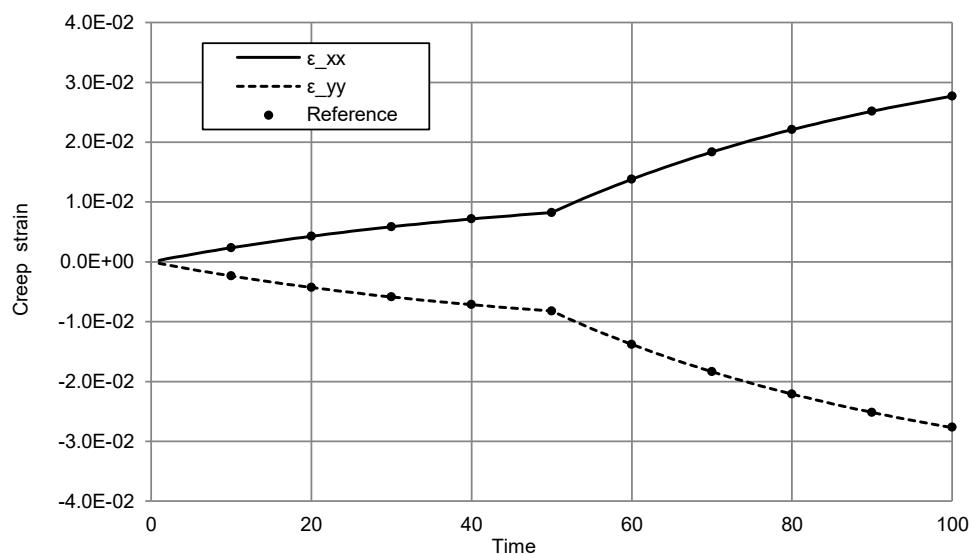


Figure 12.18.3
Creep strain of
MaterialViscoElastic18

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0024	0.0024	-0.0024	-0.0024
20	0.0043	0.0043	-0.0043	-0.0043
30	0.0059	0.0059	-0.0059	-0.0059
40	0.0072	0.0072	-0.0072	-0.0072
50	0.0082	0.0082	-0.0082	-0.0082
60	0.0138	0.0138	-0.0138	-0.0138
70	0.0184	0.0184	-0.0184	-0.0184
80	0.0221	0.0221	-0.0221	-0.0221
90	0.0252	0.0252	-0.0252	-0.0252
100	0.0277	0.0277	-0.0277	-0.0277



Figure 12.18.4
Comparison of
reference shrinkage
solution and
MaterialViscoelastic18

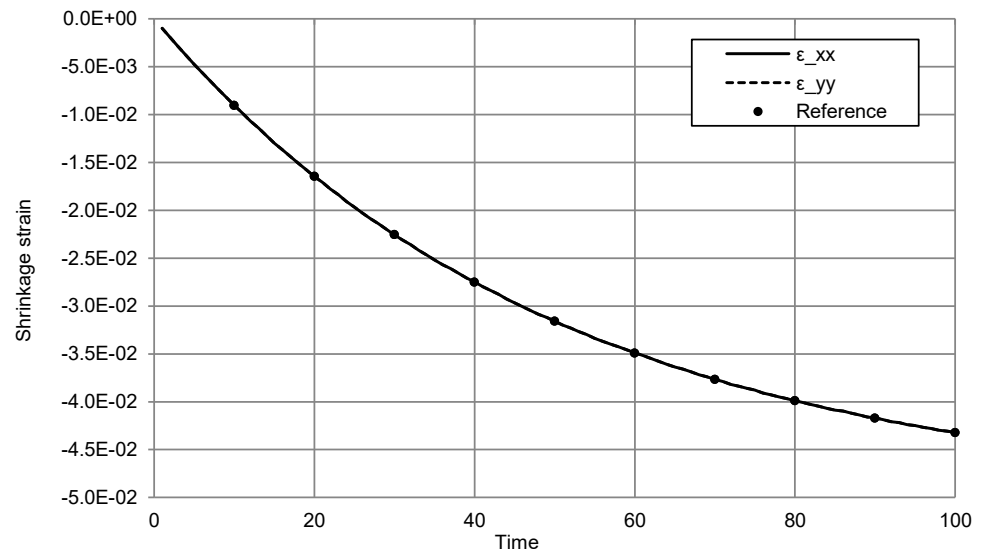


Figure 12.18.5
shrinkage strain of
MaterialViscoElastic18

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	-0.0090	-0.0091	-0.0090	-0.0091
20	-0.0165	-0.0165	-0.0165	-0.0165
30	-0.0226	-0.0226	-0.0226	-0.0226
40	-0.0275	-0.0275	-0.0275	-0.0275
50	-0.0316	-0.0316	-0.0316	-0.0316
60	-0.0349	-0.0349	-0.0349	-0.0349
70	-0.0377	-0.0377	-0.0377	-0.0377
80	-0.0399	-0.0399	-0.0399	-0.0399
90	-0.0414	-0.0417	-0.0414	-0.0417
100	-0.0432	-0.0432	-0.0432	-0.0432



Figure 12.18.6
Comparison of
reference elastic
solution and
MaterialViscoelastic18

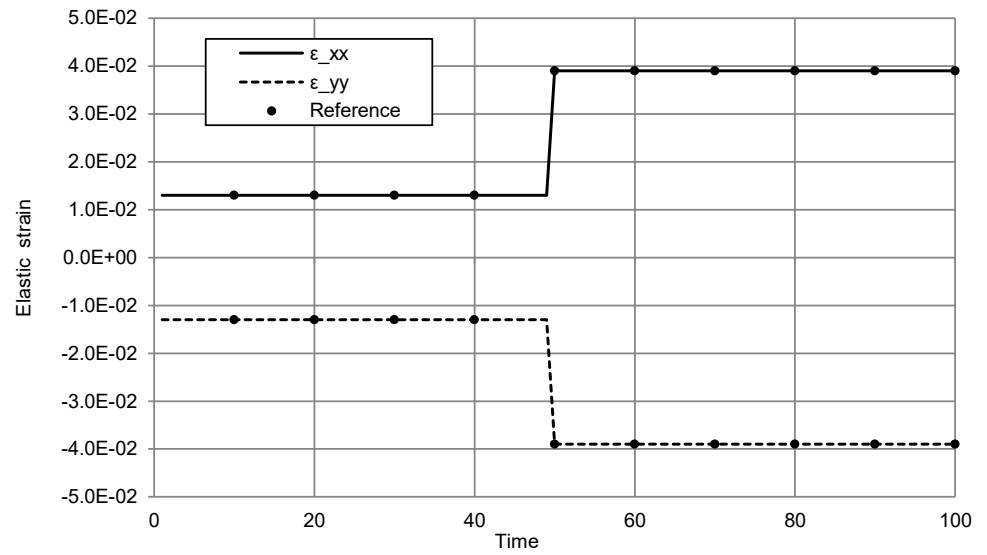


Figure 12.18.7
Elastic strain of
MaterialViscoElastic18

Time	ϵ_{xx}	Reference	ϵ_{yy}	Reference
10	0.0130	0.0130	-0.0130	-0.0130
20	0.0130	0.0130	-0.0130	-0.0130
30	0.0130	0.0130	-0.0130	-0.0130
40	0.0130	0.0130	-0.0130	-0.0130
50	0.0390	0.0390	-0.0390	-0.0390
60	0.0390	0.0390	-0.0390	-0.0390
70	0.0390	0.0390	-0.0390	-0.0390
80	0.0390	0.0390	-0.0390	-0.0390
90	0.0390	0.0390	-0.0390	-0.0390
100	0.0390	0.0390	-0.0390	-0.0390